# Endogenous Job Contact Networks and Heterogeneous Workers

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**Abstract**

This game theoretic model shows how the use of networks in the job search process amplifies the excess of supply or demand on the labor market, improves coordination, but can have opposing effects on unemployment of workers with high or low productivity. Workers can endogenously form a network that imperfectly signals productivity. Heterogenous workers choose between the uninformative formal labor market and their network as instrument for job search. We find that unemployment among workers with high productivity is always lower in presence of a network, but that a network can raise the unemployment rate for less productive workers.

**JEL classification:** J64, D85, J24
**Keywords:** search frictions, endogenous network formation, coordination frictions, heterogeneous labor

## 1 Introduction

Firms with open vacancies can usually use two different recruiting channels. A rather formal way to find applicants is to advertise on job search websites or in other media and wait for suitable candidates to apply. As a vast empirical literature shows, another method of recruiting is via informal social relationships or networking[1]. The firm's employees might recommend unemployed friends or acquaintances to their employer. This referral-based hiring can have advantages for both firms and applicants. It is often suggested that firms can improve the quality of their applicants using referral-based hiring methods. On one hand, there might a a potential correlation between characteristics of the currently employed person and her acquaintance. If the current employee fits well into the firm, a friend of her might be a comparably good match. Furthermore, from a firm's perspective, recruiting costs could be lower when using the network instead of a standard search method as the screening costs of potential candidates are supposedly lower since the network could serve as a potential source of information about the prospective employee's productivity. Furthermore, social connections among employees could facilitate smoother job-transitions because there might be a spillover effect of job-specific knowledge between a current employee and her friends. From a job-seekers perspective, there are numerous advantages of using social relationships. Most importantly, it might be easier to be employed by a firm when using a referral by a friend employed at that firm compared to sending a traditional application. It might also facilitate to even find information about firm specific vacancies, especially if there are firms who rely on their employees' referrals and to not advertise certain jobs publicly.

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[1] Ioannides and Loury (2004) and Roshchin and Markova (2004) provide an overview.

In this paper, we derive a model in which job search via formal applications and via social networks co-exist. Workers with different productivity can choose to invest effort in network formation. The network technology is based on Galeotti and Merlino (2014) and Cabrales, Calv-Armengol, and Zenou (2011): The network is stochastic, but the probability of a social connection between any two members of the population depends on their own effort as well as the efforts chosen by all members of the population. If a worker becomes unemployed, an employed friend who is connected via the network can recommend a vacancy at his own firm. In contrast, the formal labor market is always available to an unemployed worker. Thus, an unemployed worker with a referral must decide whether to use the social network or rely on the formal labour market. In our setup, traditional signalling methods in the formal labour market are completely uninformative. A network referral, on the other hand, might contain a amount of additional information as the referee knows the applicant personally. However, this does not guarantee that the referee discloses the true type of the applicant to his employer. Thus, we assume that the firm can observe the productivity of an applicant via the social network with an exogenous positive probability. We find that there are three types of potential Perfect Bayesian Equilibria (PBE): i) a zero-effort equilibrium in which not network is formed, ii) a "'separating"' equilibrium in which only workers with high productivity invest in the network, and iii) a "'pooling equilibrium"' in which both high- and low-productivity workers invest in the network in order to use it as a job search method. A zero-effort equilibrium always exists, while the existence of the other types of equilibria depends on the labor-market tightness, wages, and the cost of networking. For any parameters for which an equilibrium with positive effort exists, the unemployment rate among high-productivity workers is lower in the equilibrium with a social network. The effect of the presence of the network on workers with low productivity is ambigous. In particular, in a separating equilibrium in which only high-productivity workers invest in the network, unemployment among low-productivity workers can be higher than in absence of the network if the labour market tightness is sufficiencly low. In either case, the presence of a network is beneficial for the average match quality. Thus, the model implies that firms should encourage the formation of employee networks. A simulation using a simple welfare measure explores ranges of parameters for which the detrimental effect of the network on the low-productivity workers is compensated by the beneficial effects on high-productivity workers and firms. The results suggest that in a vast range of parameters (especially if the differences in workers' productivites are high), a network is socially desirable. Since the networking cost is a key determinant of the existence of non-zero effort equilibria, a possible policy implication could be to provide incentives for network formation by lowering these costs.

## 2 Related Literature

Starting in the 1960's with an empirical study by Rees (1966), the phenomenon of referral hiring has been investigated by labor market economists. Since then a vast amount of empirical papers on the subject has emerged.[2]

In contrast, before the 1990s, theorists have shown little interest to introduce networking as a search method in their labor market models to provide an underlying theoretical framework for the empirical findings. (one notable exception is e.g. Boorman (1975)). Later, Saloner (1985)

[2] For instance, Wei, Levin, and Sabik (2012) analyze a sample of post-doctoral researchers on the North American academic job market. The data set includes detailed information on the hiring channel used by the observed subjects and indicators as job-satisfaction and measurable scientific output. Assuming that foreign post-docs have smaller networks than domestically educated ones, they find that the match quality (measured in terms of job-satisfaction as well as in terms of the gap between potential and actual scientific output) is worse for foreign post-docs. They conclude that a network enhances the job-match quality. More recently,Dustmann, Glitz, Schnberg, and Brcker (2016) show...

shows that the information revealed by a referee about an acquainted job applicant, is the coarser the higher the number of referees, as referees use their information strategically. If there was only a single referee, he would reveal all his information to a potential employer. Nonetheless, due to the competition among referees, enough information about the applicant is revealed such that in equilibrium, firms would not change their strategies, even if the referees would reveal all their information. Saloner uses the term "network" in the sense of existing connections between job applicants and referees, and referees and potential employers, respectively.

The starting point of the recent theoretical approaches is Montgomery (1991), who introduced a model with two types, i.e. high and low ability, of employed and unemployed workers. Firms cannot observe the unemployed workers type, but are familiar with the type of their employed workers. There is a network ("a random social structure") where each employed worker knows at most one unemployed worker, a so-called neighbor in network economics, whose type is positively correlated with the employed worker's type. Unemployed workers on the contrary can have more than one neighbor. Firms are allowed to use two (mutually exclusive) hiring channels, a competitive formal labor market, where they receive direct applications, and employee referrals. Employed workers in firms hiring via job referrals pass the offer to their respective acquainted unemployed worker. The unemployed worker chooses between all of his referral offers and an application via the formal job market. In equilibrium, only firms with high ability employees hire via employee referrals, a result of the correlation between the employed workers ability and the ability of his neighbor. Consequently, firms with low ability workers prefer to hire using the normal channel. Therefore, high ability unemployed workers have a larger probability to find a job via their network which drives down the average quality on the "normal" labor market, i.e. a lemon effect. On average, better connected workers have larger wages than worse connected workers.

An obvious shortcoming of the model is the over-simplified network structure and the resulting lack of rigorous analysis of the same. Furthermore, in all of these models, the network structure already has been established and are not part of the analysis. In recent years, the rise of network economics, as introduced by Jackson and Wolinsky (1996) and Bala and Goyal (2000) provides an analytical framework to analyze the actual network structure. The first group of scholars adopting this framework in a labor market context were Antoní Calvó-Armengol and co-authors. Calv-Armengol and Jackson (2004) and Calv-Armengol and Jackson (2007) analyze the effects of information transmission about job offers between workers in a given network structure. Calv-Armengol and Jackson (2004) consider a dynamic model where workers are located on a given network. Initially, they are either employed or unemployed. At the beginning of each period, some of them randomly receive a job offer. If a job offer reaches an unemployed worker, he will use the opportunity and accept the corresponding job. If, on the other hand, an employed agent receives a job offer, he conveys the information to one randomly chosen unemployed direct neighbor. At the end of each period, any job is destroyed with an exogenous probability. In this context, unemployment status follows a Markov process. Thereby, the employment probability in the next period is positively influenced by the workers number of neighbors, since a larger number of neighbors increases the probability that at least one of them has a job to pass on. However, the effect of the number of second-degree neighbors, i.e. neighbors of direct neighbors, is ambiguous. On the one hand, if a worker's neighbors have many neighbors, the employment probability of the worker's neighbor is larger, in turn increasing the probability that he has a job to pass on. On the other hand, if he has a job to forward, the probability that he passes it on to one of his other neighbors is also increasing in the number of his neighbors. Despite this two-fold effect of second-degree neighbors, the authors show that the employment status of a worker is positively correlated with the employment status of any other worker indirectly connected to him through the network. Furthermore, Calv-Armengol and Jackson (2004) find that a worker's next period employment probability is negatively dependent on his employment duration. The intuition for this surprising result is that enduring unemployment of a worker worsens the employment situation in his network environment which in turn reflects on his own probability to find a job. Later, Calv-Armengol and Jackson (2007) generalize Calv-Armengol and Jackson (2004) by endogenizing wages and allowing for different information transmission decisions.

In contrast to assuming a fixed predetermined network structure, in Calv-Armengol and Jackson (2004) workers first form (in mutual consent) costly links to other workers. These links can be used to receive information on job offers in case of unemployment. There and in similar models, application methods do not play a role. Workers obtain job offers at random and if they are currently employed they pass them on to their neighbors. For instance, the work of Galeotti and Merlino (2014) follows this tradition. He models costly network formation with a similar labor market turnover as in Calv-Armengol and Jackson (2004), but adds a stochastic(?) component. Workers exert effort to socialize and the link formation probability increases with increased socialization effort. This formalization of the network formation process is based on a model by Cabrales, Calv-Armengol, and Zenou (2011) and it mainly enhances the tractability of the model.

Another strand of the literature focuses on the comparison between the network and a formal labor market. For instance, in the search model of Cahuc and Fontaine (2009), which is based on the Pissarides framework *(insert reference)*, workers either can use a network or a formal job market. If firms hire via the formal job market and workers use this channel to search, both, firms and unemployed workers, incur a relatively large search cost but the probability of a match is in equilibrium equal to the probability that a vacancy is opened. However, the network is taken as given and it is assumed that the degrees of all players (each player's number of links in the network) are symmetric. Workers who rely on the network as a search tool do not actively search, but wait until a job offer reaches them. If a job offer reaches an employed instead of an unemployed worker, the worker forwards it to one of his unemployed contacts. If there is no such contact, the job offer is lost. If all firms hire via the network and workers are only relying to find a job through the network, the matching probability is lower compared to the case where only the formal job market is utilized. However, workers do not incur any cost by searching through the network and the firms' hiring costs are lower. Cahuc and Fontaine (2009) find that the situation where everybody uses the network as a search method is always an equilibrium while the situation where everybody searches via formal search methods is an equilibrium only for low relative costs of this search method and a relatively loose social network. They also find that the intensity of networking might be inefficient.

Casella and Hanaki (2006) and Casella and Hanaki (2008) focus on costly signalling as an information transmission device of a worker's productivity. Their main finding in the latter paper is that the availability of the signal, and, therefore, more detailed information on an applicant's productivity than transmitted through the network, does not necessarily weaken a firms reliance on the network if links are without costs. That is, in their model equilibria exist in which firms use referral hiring via the network although perfect information on the worker's type would be available via the signal. However, the productivity signal is not available prior to the firm's decision whether to hire a network applicant or not, but only afterwards signal is revealed to the firm, provided it successfully acquired the signal. In Casella and Hanaki (2006), with higher costs of networking, signalling surpasses the network in terms of the information revealed about the worker's productivity. Therefore, the assumption that the relationship between signalling and the network is more of a substitutive nature is in strong contrast with this paper, in which the signal precisely conveys information about the network applicant's productivity.

# 3 Model Setup

Consider a large labor market with a population $\mathcal{M}$ of risk neutral workers and a population of firms that each employ exactly one worker. Initially, there are $|\mathcal{M}| = M$ workers and $N = M$ firms whereby each firm employs exactly one worker. Workers are heterogeneous in their productivity, whereby each worker's productivity is high ($p^h$) with probability $r$ and low ($p^l$) with probability $(1 - r)$. The productivity is observable only to the workers themselves and to their current employers.

The time horizon is one period. Each worker who is employed at the end of this period earns an exogenously given wage $w < p^l < p^h$. Hence, firms would prefer to employ high productivity workers whenever this is possible, but if they do not have another choice they also would employ low productivity ones. Unemployed workers receive unemployment benefit $b < w$.

The model has four phases (see figure xxx): *Insert timeline or probably game tree*

1. **Phase I: Networking**

   The network formation technology is adapted from Cabrales, Calv-Armengol, and Zenou (2011) and Galeotti and Merlino (2014). In phase I, each worker $i \in \mathcal{M}$ of type $p \in \{h, l\}$ chooses a networking effort $e_{ip} \geq 0$. Thereby, worker $i$ might choose a different value if he is of the high-productivity type compared to a worker with low productivity. The effort spent on networking follows a linear cost function, $C(e_{io} = ce_{ip}$, whereby $c > 0$ is the marginal cost of effort.

   The network is random, but the networking efforts affect the probability of link formation. In particular, a link between worker $i$ and worker $j$ is formed with probability

$$g_{ij} = \min \left( \frac{e_i e_j}{\kappa(e_1, \ldots, e_M)}, 1 \right) \tag{1}$$

   with $\kappa(e_1, \ldots, e_M) = \begin{cases} \sum_{k=1}^{M} e_k, & \text{if} \sum_{k=1}^{M} e_k > 0 \\ 1, & \text{otherwise} \end{cases}$

2. **Phase II: Exogenous Shock**

   Each firm is hit by an exogenous shock with probability $0 < s < 1$. This shock destroys the firm-worker-match, the firm leaves the market and the worker becomes unemployed. Each firms that is still in the market creates a new vacancy with the exogenous probability $0 < v < 1$. Denote by $\theta$, the labor market tightness, i.e. the ratio of unemployed workers and newly created vacancies,

$$\theta = \frac{(1 - s)v}{s}.$$

3. **Phase III: Applications**

   Matches between unemployed workers and firms with a vacancy can be generated on the formal labor market or via a network based informal labor market. Unemployed workers can send exactly one application, either as a referral through the network or on the formal labor market. However, an application through the network is only available if the unemployed worker has a friend who offers him a referral. In contrast, the formal labor market is open to everyone. If an unemployed worker is offered by at least one of his friends a referral, he can randomly choose one of these offers, or apply through the formal labor market. If he is not offered any referral, he must apply through the formal labor market.

Each worker who is employed at a firm with a vacancy and has at least one unemployed friend, randomly chooses one of his unemployed friends and offers him a referral. If the friend agrees to use this referral, the employed worker recommends his friend to his firm. If the unemployed worker refuses the offer, the firm will not receive a network application.

4. **Phase IV: Hiring**

   Firms can either have one or zero network applicants. Before they make the hiring decision, firms learn the true type of a network applicant with probabilities $\gamma_h$ and $\gamma_l$, respectively. Firms decide on whether to hire the network applicant. A firm with applicants via both channels might find it beneficial to decline the network applicant and rely on the formal labor market, just like all firms without a network applicant. However, a firm that realizes that it has no formal applicant will always hire the network applicant. A declined network applicant will stay unemployed for the remainder of the period.

   On the formal labor market, workers and firms meet according to a matching technology with search frictions. Hence, there might be firms without formal applicants and firms with several applicants. On the formal labor market, the productivity of a worker cannot be observed by a firm prior to employment. Each firm that uses the formal labor market and has at least one applicant, randomly chooses and hires one of these workers. Workers who are not selected stay unemployed and vacancies without applicants remain unfilled.

**Discussion**

Reaching a pairwise stable network or a Nash network as in Jackson and Wolinsky (1996) or Bala and Goyal (2000) would require much coordination among players which seems unrealistic in a very large population. Thus, we assume that neither workers nor firms can observe the productivity of an unkown agent. Hence, socialization effort cannot be directed towards any particular other worker. In our a framework, the approach of Galeotti and Merlino (2014) seems better suited to be both tractable and realistic. Following Cabrales, Calv-Armengol, and Zenou (2011), they assume a random network that is build according to strategically chosen networking efforts.

We choose not to model the referees' degree of loyalty to their friends and their employers explicitly, because many scenarios are conceivable. For instance, on the one hand, if referees are more loyal to their employers than to their friends, they would pass the referral to a high productivity friend. On the other hand, if they are more loyal to certain type of friends (e.g. homophily), they would select this type of friend without considering the interest of the firm. Additionally, non-loyalty based incentives for the referees might exist: in order to be able to free-ride on the high productivity worker, they could prefer high-productivity co-workers over low-productivity co-workers. Conversely, in order to excel with an own performance above the firm average, they could prefer low productivity co-workers. Our choice of a random referral is neutral, and can be seen as an "average" of these opposing possible incentives.

Obviously, a friend of a low productivity worker would not tell the firm directly that the person he recommends is of low productivity. The signal that the firm receives with probability $\gamma_h$ or $\gamma_l$, however, is supposed to reflect the fact that the firm knows its own employee and can decrypt up to a certain extent the informational content of the recommendation.

# 4 Preliminary analysis of the model

We will use Weak Perfect Bayesian Equilibrium as equilibrium concept and focus on symmetric equilibria. Note that ex-ante, each worker $i \in \mathcal{M}$ has a pair of phase I strategies, an effort level $e_{ih}$ in case he is of the high-productivity type and $e_{il}$ in case he is of the low-productivity type. Hence, a phase I strategy of each worker is a tuple $e_i(e_{ih}, e_{il}$. In symmetric equilibria, all workers of the same type choose the same networking effort in phase I and the same action in any situation in which they have to take an equivalent decision. In particular, in phase I, all workers of the high-productivity type choose the same networking effort $e_{ih} = e_h \forall i \in \mathcal{M}$ and all workers of the low-producitivty type choose the same networking effort $e_{il} = e_l \forall i \in \mathcal{M}$. In phase III, either all high-productivity (low-productivity) workers who have a referral offer choose to accept this offer or not. Furthermore, in phase IV, either all firms who have a network applicant and receive the high productivity signal (the low-productivity signal, no signal) choose to employ their respective network applicant or not. As we focus on this specific type of equilibria, we also focus on deriving each worker $i$'s payoff function, for his type $p \in \{h, l\}$, given he chooses a particular effort level $e_{ip}$ and all other workers choose a symmetric effort level $e_h$ or $e_l$ according to their types. Following Reny (1999), given a fixed, symmetric action profile for all players at any subsequant phases, we will denote the corresponding payoff functions as "'diagonal payoff functions"' for type $p \in \{h, l\}$. We restrict the set of availably networking efforts in phase I to[3]

$$e_{ip} \in [0, sw/c]. \tag{2}$$

Note that this constraint is without loss of generality, because whenever the probability to find a job via the network is strictly less than 1, employing a networking effort greater than $sw/c$ would yield a benefit lower than $sw$ (the expected wage if a new job is found with probability one) at a cost greater than $c(sw/c) = sw$. Hence, rational workers would never choose a networking effort larger than $sw/c$.

In phase I, each worker decides on his effort level, i.e. he chooses $e_i$ $e_i = (e_{ih}, e_{il})$. As we deal with an (infinitely) large population, fraction $r$ of the population can be considered of the high productivity type and fraction $(1-r)$ of the population of the low productivity type. Thus, $rM$ workers choose effort level $e_h$ and $(1-r)M$ workers choose effort level $e_l$. In the following paragraph, we derive the probability for an unemployed worker to find a network referral.

## 4.1 Network Matching

Consider a worker $i$ with type $p \in \{h, l\}$. He invests effort level $e_{ip}$ and all other workers invest either effort level $e_h$ or $e_l$ according to their individual types. Then, according to (1), the probability to be connected to an arbitrary worker ot type $\overline{p} \in \{h, l\}$ is

$$g_{ip\overline{p}} = e_{ip} \cdot \frac{e_{\overline{p}}}{re_h + (1-r)e_l)M} \text{ if } e_h > 0 \text{ or } e_l > 0 \tag{3}$$

and $g_{ip\overline{p}} = 0$ if both $e_h = 0$ and $e_l = 0$. For $e_{ip} = e_h$ ($e_{ip} = e_l$) we denote the resulting symmetric link intensities by $g_{hp}$ and $g_{pl}$. We now derive the probability that a worker $i$ of type $p \in \{h, l\}$ chooses a particular other worker $j$ to give him a referral, conditional on $i$ and $j$ being connected to each other. We denote this probability by $\hat{\alpha}_p(e_h, e_l)$. Given the networking technology, the distribution of the number $i$'s unemployed friends in $\mathcal{M} \setminus \{i, j\}$ is binomial with expectation $(rg_{hp} + (1-r)g_{pl})s(M-2)$. Hence,

$$\hat{\alpha}_p(e_h, e_l) = \sum_{x=0}^{M-2} \binom{M-2}{x} (rg_{hp} + (1-r)g_{pl})s)^x (1 - (rg_{hh} + (1-r)g_{pl})s))^{M-2-x} \frac{1}{x+1} \tag{4}$$

[3]This restriction ensures a compact strategy space, such that the game is a compact game in the sense of Reny (1999). This allows us to prove the existence of a symmetric equilibrium.

As $M$ approaches infinity and $e_p > 0$, $\hat{\alpha}_p(e_h, e_l)$ converges to

$$\lim_{M\to\infty} \alpha_p^s(e_h, e_l) = \frac{1 - \exp[-e_p s]}{e_p s}. \tag{5}$$

As this expression only depends on the effort of worker $j's$ own type $p$, we will henceforth skip the effort of the opposite type in the argument of the $\alpha$-functions. If $e_p = 0$, the probability $\alpha_p(e_p) = 0$.

Using $\alpha_p(e_p)$, we can now define the probability $\bar{\epsilon}_{ip}(e_{ip}, e_h, e_l)$ that an unemployed worker $i$ of type $p \in \{h, l\}$ would find at least one other worker in the network who is able and willing to give him a job referral in case he becomes jobless. This is the counter-probability of not finding any such referral. For $e_h > 0$ or $e_l > 0$,

$$\bar{\epsilon}_{ip}(e_{ip}, e_h, e_l) = 1 - (1 - \theta s(r g_{ih} \alpha_h(e_h) + (1-r) g_{il} \alpha_l(e_l)))^{M-1}.$$

If $e_h = 0$ and $e_l = 0$,

$$\bar{\epsilon}_{ip}(e_{ip}, 0, 0) = 0. \tag{6}$$

When $M$ approaches infinity, the limit of $\bar{\epsilon}_{ip}(e_{ip}, e_h, e_l)$ is

$$\bar{\epsilon}_{ip}(e_{ip}, e_h, e_l) \equiv 1 - \exp[e_{ip} f(e_h, e_l)] \text{ with} \tag{7}$$

$$f(e_h, e_l) = \begin{cases} \dfrac{(-1 + r\exp[-e_h s] + (1-r)\exp[-e_l s])\theta}{r e_h + (1-r) e_l} & \text{if } e_h > 0 \text{ or } e_l > 0 \\ 0 & \text{else} \end{cases} \tag{8}$$

In the analysis of phases III and IV, we will denote by $\epsilon_h(e_h, e_l)$ $(\epsilon_l(e_h, e_l))$the probability for a high (low) productivity worker to find a network referral, when choosing the same effort level $e_h$ $(e_l)$ as all other workers of their type, given all workers of the other type also choose the same effort level $e_l$ $(e_h)$.

## 4.2 Formal Labor Market Matching

We define $V_A$ the number of firms present on the formal labor market and by $M_{Ap}$ the number of workers of type $p \in \{h, l\}$ present on the formal labor market. Firms present on the formal labor market either did not receive any network applicant in phase III or have refused their network applicant at the beginning of phase IV. Similarly, workers who participate in the the formal labor market either did not find a job referral via the network or have voluntarily decided not to use their referral in phase III. Recall that we assume that a worker who was refused during a network job interview cannot send any application on the formal labor market. Hence, the values of $M_{Ap}$ and $V_A$ depend on the effort levels chosen by workers in phase I (as this determines the individual probabilities to have a net work applicant or referral), and on the firms' decision in phase IV (whether to hire their network applicants). We will examine this relationship in more detail in section 5.1 below. Denote by $\vartheta$ the tightness on the formal labor market, i.e. the ratio of vacancies and unemployed workers present on the formal labor market,

$$\vartheta \equiv \frac{V_A}{M_{Ah} + M_{Al}}. \tag{9}$$

Each worker writes exactly one application, and each firm that receives at least one application randomly chooses exactly one of its applicants. Thus, the number of vacancies that are filled on the formal labor market is equal to[4]

$$M_f = V_A\left(1-\left(1-\frac{1}{V_A}\right)^{M_{Ah}+M_{Al}}\right) \tag{10}$$

The term in brackets is the individual probability of a vacancy to receive at least one application. Another way to write this number is (see Albrecht, Gautier, and Vroman (2003))

$$M_f = (M_{Ah}+M_{Al})\left[\frac{V_A}{M_{Ah}+M_{Al}}\left(1-\frac{1}{V_A}\right)^{M_{Ah}+M_{Al}}\right] \tag{11}$$

The term in square brackets is the individual probability of a network applicant to be selected by the firm he sends his application to. As $M$ approaches infinity, the individual probability of a firm on the formal labor market to fill its vacancy is

$$\lim_{M\to\infty} = \left(1-\left(1-\frac{1}{V_A}\right)^{M_{Ah}}+M_{Al}\right) = 1-\exp\left[-\frac{M_{Ah}+M_{Al}}{V_A}\right]. \tag{12}$$

Recalling that $\vartheta$ denotes the tightness of the formal labor market, we can rewrite the probability of a firm to fill its vacancy conditional on participating in the formal labor market (12) as function of theta,

$$\phi(\vartheta) \equiv 1-\exp[-1/\vartheta]. \tag{13}$$

As $M$ approaches infinity, the individual probability of a worker to find a job on the formal labor market, conditional on having entered it, is

$$\lim_{M\to\infty}\frac{V_A}{M_{Ah}+M_{Al}}\left(1-\left(1-\frac{1}{V_A}\right)^{M_{Ah}+M_{Al}}\right) = \frac{V_A}{M_{Ah}+M_{Al}}\left(1-\exp\left[-\frac{M_{Ah}+M_{Al}}{V_A}\right]\right) \tag{14}$$

This can also be expressed as a function of $\vartheta$,

$$\psi(\vartheta) = \vartheta(1-\exp[-1/\vartheta] = \vartheta\phi(\vartheta) \tag{15}$$

Hence, the formal and the informal labor markets are structured in a similar way. Indeed, $(e_p s)^{-1}$ in $\alpha_p$ (see (5)), i.e. the probability of a worker of type $p$ to be chosen for a referral by a connected worker is comparable to the role of $\vartheta$ in $\psi(\vartheta)$, i.e. the probability of a worker to be selected by the firm he sends an application to on the formal labor market. Thus, a connection in the network has similar properties as an application on the formal labour market. that a worker is chosen by the firm he sends an application to on the formal labor market. For later reference, consider the case when none of the workers receives any network referral. Then the number of applicants and firms on the formal labor market coincide with their equivalents in the whole economy, i.e. $V_A = (1-s)vM = s\theta M$, $M_{Ah} = rsM$, and $M_{Al} = (1-r)sM$. In that case, the tightness of the formal labor market is $\vartheta = (s\theta M)/(rsM+(1-r)sM) = \theta$. We denote the probability of a firm to hire a worker in the case when $e_h = e_l = 0$ by $\phi^0$ and the probability

[4] See Albrecht, Gautier, and Vroman (2003). Note that the corrections to the matching function as presented in Albrecht, Tan, Gautier, and Vroman (2004) do not apply to our paper, because of our assumption that workers can send only one application.

of a worker to be hired by $\psi^0$, i.e.

$$\phi^0 = \phi(\theta) \quad (16)$$
$$\psi^0 = \psi(\theta) \quad (17)$$

**Lemma 1.** *The function $\psi(\vartheta)$ is strictly increasing in $\vartheta$ and the function $\phi(\vartheta)$ is strictly decreasing in $\vartheta$.*

[Proof]
The proof and all further proofs not directly provided after a result are relegated to the appendix.

In order to understand Lemma 1, recall that the larger $\vartheta$, the more vacancies per jobless worker are in the market. Hence, from a job-seeker's point of view, an increase in $\vartheta$ renders the formal labor market less competitive and it becomes easier to find a firm with a vacancy for a given worker. By the same argument, an increase in $\vartheta$ makes it harder for firms to find a jobless worker.

# 5 Equilibria of the Model

## 5.1 General Characterization of Equilibria

We solve the model by backward induction. First, we examine the firms' and the workers' decisions in phase III and IV to derive a set of possible equilibria.

**Firms' decisions in phase IV**

Consider a job-interview in phase IV. If the applicant is of the high-productivity (low-productivity) type, his true type is disclosed to the firm with probability $\gamma_h$ ($\gamma_l$). Upon receiving the high-productivity signal, the potential employer will hire the network applicant for sure because he then will hire a high-productivity worker with certainty, whereas on the formal labor market, there is a strictly positive probability to hire a low-productivity worker. In contrast, if the low-productivity signal is received, the firm will not hire the network applicant directly because on the formal labour market, there is a strictly positive probability to hire a high-productivity worker. However, in case no worker applies via the formal labour market, the firm will still prefer to hire the low-productivity network applicant to an unfilled vacancy. If the firm does not receive any signal, the decision depends on the firm's beliefs about the proportion of high- and low-productivity workers in the networn and on the formal labor market. The firm will hire the network candidate directly if the probability of this candidate to be of the high-productivity type (conditional on being connected to the firm's employee and conditional on the firm not having received any signal) is larger than the probability that an arbitrary candidate present on the formal labor market is of the high-productivity type. We will distinguish two types of pure strategies for a firm that receives a network applicant without a productivity signal: to accept the applicant directly, or to search for another applicant on the formal labor market. If the latter strategy is not sucessful, the firm will hire the network applicant without signal.

**Workers' decisions in phase III**

As we restrict our analysis to symmetric, pure strategy equilibria, workers can expect wo different types of firms' behavior in phase IV: Either they hire the network applicant directly in absence of a signal, or the do not. In the former case, high-productivity workers will always use their network referral. In the latter case, they will use their network referral if and only of

$$\gamma_h \geq \frac{\psi(\vartheta) - (1 - \phi(\vartheta))}{\phi(\vartheta)} \tag{18}$$

Low-productivity workers who expect firms to hire network applicants in case they do not receive a signal choose to use their network referral in phase III if and only if

$$\gamma_l \leq \frac{1 - \psi(\vartheta)}{\phi(\vartheta)} \equiv \overline{\gamma}(\vartheta) \tag{19}$$

The following lemma identifies an upper and a lower bound to $\overline{\gamma}(\vartheta)$.

**Lemma 2.** *For all* $0 < \vartheta < \infty$*,* $\overline{\gamma}(\theta)$ *is monotonically decreasing in* $\vartheta$*. Furthermore,* $1/2 < \overline{\gamma}(\vartheta) < 1$ *and*

$$\begin{aligned} &\textit{if} \quad \vartheta \leq 1, \quad \overline{\gamma}(\vartheta) \in [(\exp[1] - 1)^{-1}, 1[ \\ &\textit{if} \quad \vartheta > 1, \quad \overline{\gamma}(\vartheta) \in ]0.5, (\exp[1] - 1)^{-1}[. \end{aligned}$$

[Proof]
An increase in $\vartheta$ increases the low-productivity worker's probability to find a job through the formal labor market $\psi(\vartheta)$ and therefore the opportunity cost of taking the network referral. Hence, a tighter formal labor market reduces the threshold $\overline{\gamma}(\vartheta)$. Note that an increase in $\vartheta$ also reduces the firm's probability to find an applicant in the formal labor market and therefore increases the benefits of accepting the network referral. However, we show that the latter effect is always smaller than the effect of the increase opportunity cost.
If $\gamma_l \leq 0.5$, low-productivity workers always choose to use the network in phase III. Furthermore, if $\vartheta \leq 1$, low-productivity workers always choose to use the network in case $\gamma_l < (\exp[1] - 1)^{-1}$. By contrast, if $\vartheta > 1$ and $\gamma_l > (\exp[1] - 1)^{-1}$, low-productivity workers never choose to use the network in phase III. If a low-productivity worker expects firms not to hire network applicants directly in case they do not receive a signal, the firm does not receive any application on the formal labor market with probability $(1 - \phi(\vartheta))$. Hence, a low-productivity worker chooses to use his network referral in phase III if and only if

$$1 - \phi(\vartheta) \geq \psi(\vartheta) \tag{20}$$

**Lemma 3.** *If, according to low-productivity worker i's expectations, none of the firms hire workers via the network without a productivity signal, then i does not use his network referral in phase III. That is, for* $\vartheta > 0$*, inequality* (20) *is never fulfilled.*

[Proof] Workers only excert effort in phase I if the intend to use the potential network referral in phase III. If workers do not intend to use the network, they will not excert effort and thus cannot receive any network offer.

**Lemma 4.** *In any possible equilibrium with* $e_{ip} > 0 \; \forall \; i \in \mathcal{M}$ *and at least one type* $p \in \{h, l\}$*, on the equilibrium path, any worker chooses to use the network if and only if he is of a type with* $e_{ip} > 0$*.*

[Proof]

**Corollary 1.** *All workers who have a network referral will use it on any equilibrium path.*

Combining the results from Lemma 3 and Lemma 4, we can observe that there will never be a symmetric pure strategy equilibrium in which low-productivity workers exert any strictly

| | **Numbers of participants on the formal market** | $\vartheta$ |
|---|---|---|
| **Pooling I**<br>All firms: hire<br>when no signal<br>All workers: network | $M_{Ah} = rs(1-\epsilon_h)M$<br>$M_{Al} = (1-r)s(1-\epsilon_l)M$<br>$V_A = Ms(\theta - r\epsilon_h - (1-r)\epsilon_l(1-\gamma_l))$ | $\frac{\theta - r\epsilon_h - (1-r)(1-\gamma_l)\epsilon_l}{1-r\epsilon_h-(1-r)\epsilon_l}$ |
| **Separating**<br>Firms: hire<br>$p_l$: formal<br>$p_h$: network | $M_{Ah} = rs(1-\epsilon_h)M$<br>$M_{Al} = (1-r)sM$<br>$V_A = Ms(\theta - r\epsilon_h$ | $\frac{\theta - r\epsilon_h}{1-r\epsilon_h}$ |
| **Pooling II**<br>$p_l$: formal<br>$p_h$: formal | $M_{Ah} = rsM$<br>$M_{Al} = (1-r)sM$<br>$V_A = Ms\theta$ | $\theta$ |

Table 1: Number of Participants in the Formal Labor Market and $\vartheta$ in possible Equilibrium Outcomes

positive effort level in phase I and firms do not directly accept workers in phase IV in case the do not receive any productivity signal. Stressing consistency of the beliefs, it is a necessary condition for a symmetric PBE that, on the equilibrium path, firms always directly accept workers in the no-signal case in phase IV. To see this, suppose firms do not accept any network applicant in the case of no signal. Then only high-productivity workers possibly play a strictly positive effort level. However, since low-productivity workers always play zero-effort in phase I if they expect firms to behave this way in phase IV, any network applicant must be of the high-productivity type. If that is the case, firms do not reject network applicants in the no-signal case, which contradicts the assumption that they reject any network applicant in absence of a signal. Hence, on the equilibrium path, in any symmetric pure strategy equilibrium, firms will always accept any network applicant in the no-signal case. Therefore, the set of possible equilibria can be restricted as follows.

**Proposition 1.** *Only the following three cases of symmetric pure strategy PBE outcomes are possible:*

1. **Zero Effort:** $e_i = (0,0)$ *for all workers.*
2. **Separating:** $e_i = (e_h, 0)$, $\forall i$, *with* $e_h > 0$, *and all high productivity workers who have a network referral send an application through the network.*
3. **Pooling:** $e_i = (e_h, e_l)$, $\forall i$, *with* $e_h > 0$ *and* $e_l > 0$, *and all workers who have a network referral send an application through the network.*

Table 1 summarizes the numbers of applicants and vacancies on the formal labor market, given different symmetric pure strategy profiles of workers and the firms' symmetric best responses to it.

Substituting the numbers of workers and firms remover from the labor market by the network into (9), we obtain, for any possible symmetric equilibrium, $\vartheta$ as function of $e_h$ and $e_l$,

$$\vartheta(e_h, e_l) \quad = \quad \frac{\theta - r\epsilon_h(e_h, e_l) - (1-r)(1-\gamma_l)\epsilon_l(e_h, e_l)}{1 - r\epsilon_h(e_h, e_l) - (1-r)\epsilon_l(e_h, e_l)} \tag{21}$$

Note that even if the initial labor market tightness is very close to zero, $\vartheta$ will always remain strictly positiv, that is, there is always a strictly positive probability that some vacancies remain unfilled that do not receive any application due to coordination frictions. The following lemma summarizes this observation.

**Lemma 5.** $\vartheta > 0$ *on any symmetric pure strategy equilibrium path.*

[Proof]
The observations of this chapter allow us to re-write the firms' and the workers' matching probabilities on the formal labor market $\psi(\vartheta)$ and $\phi(\vartheta)$ as functions of the symmetric effort levels chosen in phase I.
Denote by

$$\psi(e_h, e_l) = \psi(\vartheta(e_h, e_l)) \tag{22}$$

$$\phi(e_h, e_l) = \phi(\vartheta(e_h, e_l)) \tag{23}$$

Hence, for the remainder of the paper, if $\psi$ or $\phi$ are used with a single argument, the formulae from (13) and (15) apply. If $\psi$ or $\phi$ are written as functions of two arguments, they refer to the formulae from (22) and (23).

### 5.2 Zero Effort Equilibrium

In the following sections, we will analyse the equilibrium candidates from Proposition 1 in more detail. We start our analysis with the zero-effort case (case one of Proposition 1.

**Lemma 6.** *A situation where* $e_i = (0, 0)$ $\forall$ $i \in \mathcal{M}$ *is always a PBE, regardless of the play off the equilibrium path.*

If nobody else invests any effort into socialization, it does not pay off for an individual worker to exert effort himself, because he will not be able to make any connections. As effort is costly, the unique best response to a situation where nobody else invests any effort is therefore not to socialize, either. This result can be found in the literature on deterministic endogenous link formation as well, where in two-sided link formation models, the empty network has been found to be always a Nash equilibrium. Although it does not seem to be entirely plausible that workers refrein from forming social connections at all, the economy without exchange of job-referrals through the worker's individual network provides a suitable benchmark for our analysis. Furthermore, this equilibrium coincides with the world assumed in the standard labor market search model in the spirit of Pissarides (2000), which therefore can considered as a special case of our model.

**Lemma 7.** *In the zero-effort equilibrium, the overall unemployment rate is equal to* $s(1 - \psi^0(\theta))$ *and a total of* $s(\theta - \psi^0)M$ *vacancies remain unfilled. Furthermore, the average probability of a filled job to be occupied by a high-productivity worker is* $r$*.*

[Proof]

### 5.3 Networking Equilibria

We will now analyze equilibrium candidates in which at least high-productivity workers play a strictly positive effort level. As shown before, from all situations where at least one type of worker plays a strictly positive, symmetric effort level, only those qualify for an equilibrium in which firms directly accept network applicants if they receive the high-productivity signal or no signal at all, and prefer to search on the formal market if they receive a sow-productivity signal. If all low-productivity workers have chosen zero effort in phase I, i.e. in all "'separating"' situations, this is automatically the best response for any firm in phase IV. However, if low-productivity workers set strictly positive effort levels in phase I, it is possible that firms rather deviate to not directly accepting network applicants in case there is no productivity signal. Hence, for "'pooling"' cases, we need to check whether it is the best response for the firm to accept any network applicant in absence of a low-productivity signal. We refer to this as the firms' participation constraint in a pooling equilibrium. As phase III was already generally analyze in the previous section, we now discuss the high-productivity workers' decision in phase I.

Denote by $\rho_{ip}(e_{ip,e_h,e_l})$ the probability that a jobless worker $i$ with productivity $p \in \{h, l\}$ is employed at the end of the period. High-productivity workers find a job through the network with probability $\epsilon(e_{ih}, e_h, e_l)$, that is, the probability to receive a referral offer. With probability $1 - \epsilon(e_{ih}, e_h, e_l)$ they have to send an application on the formal labor market. In that case, the firm accepts their application with probability $\psi(e_h, e_l)$. Thus,

$$\rho_{ih}(e_{ih,e_h,e_l}) \quad = \quad \epsilon_{ih}(e_{ih}, e_h, e_l) + (1 - \epsilon(e_{ih}, e_h, e_l))\psi(e_h, e_l). \tag{24}$$

Low-productivity workers with a network referral directly receive a job offer at the beginning of phase IV only if their productivity is not disclosed to the form, that is with probability $(1 - \gamma_l)$. However, in case firms receive the low-productivity signal, due to labor market frictions, with probability $(1-\phi(e_h, e_l))$ no worker applies to their vacant job on the formal labor market. Hence, for unemployed low-producitivity workers who have a network referral offer, the probability to be employed through the network is equal to $(1 - \gamma_l) + \gamma_l(1 - \phi(e_h, e_l)) = 1 - \gamma_l\phi(e_h, e_l)$. The probability to find a job thorugh the labor market in case the worker does not have a network referral is analogous to that of the high-productivity workers. Thus, the probability to be employed at the end of the period is

$$\rho_{il}(e_{il,e_h,e_l}) \quad = \quad \epsilon_{il}(e_{il}, e_h, e_l)(1 - \gamma_l\phi(e_h, e_l)) + (1 - \epsilon(e_{il}, e_h, e_l))\psi(e_h, e_l). \tag{25}$$

In the following, for all $p \in \{h, l\}$, we will denote by $\rho_p(e_h, e_l)$ the probability that a jobless worker of type $p$ finds a job if he plays the same effort level as all other workers of his type, i.e. $e_{ip} = e_p$.

The diagonal payoff function of worker $i$ of type $p \in \{h, l\}$ can be written as

$$\Pi_{ip}(e_{ip}, e_h, e_l) = (1 - s)w + sw\rho_{ip}(e_{ip}, e_h, e_l) - ce_{ip} \tag{26}$$

**Lemma 8.** *$\Pi_{ih}(e_{ih}, e_h, e_l)$ is strictly concave in $e_{ih}$ whenever $e_h > 0$ or $e_l > 0$. Whenever $e_h > 0$ or $e_l > 0$, $\Pi_{il}(e_{il}, e_h, e_l)$ is strictly concave in $e_{il}$ if and only if $\gamma < \overline{\gamma}(\vartheta(e_h, e_l))$ and convex if and only if $\gamma_l \geq \overline{\gamma}(\vartheta(e_h, e_l))$.*

[Proof] Hence, the payoff functions of both types of workers are concave whenever $e_h$, $e_l$ and $\gamma_l$ are such that the low-productivity workers' phase III participation constraint (19) is fulfilled. Note that by Lemma 2, this is always the case whenever $\gamma_l \leq 0.5$.

For high-productivity workers, the first order condition for a payoff maximum is

$$\frac{\partial \epsilon_{ih}(e_{ih}, e_h, e_h)}{\partial e_{ih}} \cdot (1 - \psi(e_h, e_l))\, sw = c \tag{27}$$

We substitute (6) for $\epsilon_{ih}(e_{ih}, e_h, e_h)$. In case $e_h$ or $e_l$ are strictly positive, the first order condition is

$$-\exp[e_{ih}f(e_h, e_l)]f(e_h, e_l)(1 - \psi(e_h, e_l))sw = c \tag{28}$$

Note that the term on the left hand side is positive because $f(e_h, e_l)$ is negative. By Lemma 8, any solution to this equation is a maximum of the high-productivty worker's payoff maximization problem and thus corresponds to a best response to $e_h$ and $e_l$. Therefore,

$$BR_{ih}(e_h, e_l) = \max\left(\frac{\ln[\frac{sw}{c}] + \ln[-f(e_h, e_l)] + \ln[1 - \psi(e_h, e_l)]}{-f(e_h, e_l)}, 0\right). \tag{29}$$

We solve a low-productivity worker's maximization problem analogously. The first order condition for a maximum of the objective function is

$$\frac{\partial \epsilon_{il}(e_{il}, e_h, e_h)}{\partial e_{il}} \cdot (1 - \psi(e_h, e_l) - \gamma_l \phi(e_h, e_l))\, sw = c \tag{30}$$

By Lemma 8, the solutions to this equation constitute maxima of the objective function if and only if $\gamma_l < \overline{\gamma}(\vartheta(e_h, e_l))$. Hence, a low-productivity worker's best response to a given symmetric strategy profile of the other workers with $e_h > 0$ or $e_l > 0$ is

$$BR_{il}(e_h, e_l) = \begin{cases} \max\left(\frac{\ln[\frac{sw}{c}] + \ln[-f(e_h, e_l)] + \ln[1 - \psi(e_h, e_l) - \gamma_l \phi(e_h, e_l)]}{-f(e_h, e_l)}, 0\right) & \text{if } \gamma_l < \overline{\gamma}(\vartheta(e_h, e_l)) \\ 0 \text{ if } \gamma_l \geq \overline{\gamma}(\vartheta(e_h, e_l)) & \end{cases} \tag{31}$$

Any non-zero-effort-equilibrium is characterized by the following system of equations which must hold in addition to the firms' participation constraints (to accept any network applicant if there is no signal of low-productivity) and the low-productivity workers' phase III participation constraint in pooling equilibria (to send a network application whenever there is a job referral).

$$e_h = BR_{ih}(e_h, e_l) \tag{32}$$

$$e_l = BR_{il}(e_h, e_l) \tag{33}$$

**Proposition 2.** *If firms accept any network applicant in the no-signal case in phase IV and if*

$$\gamma_l < \frac{1 - \psi^0 - c/(s^2 w\theta)}{\phi^0} \text{ and } \gamma_l < 0.5, \tag{34}$$

*there exists at least a tuple* $(e_h, e_l)$ *with* $e_h$ *or* $e_l$ *such that the system* (32)-(33) *is fulfilled.*

[Proof]

The proof is given in the appendix. It relies on a well-known existence theorem by Reny (1999). The theorem states sufficient conditions for the existence of at least one equilibrium. As we already know that our original game has always a zero-effort equilibrium, we modify our game by altering the values of the payoff functions to a negative value when all players of all types play zero effort. Then we use the theorem to find sufficient conditions for the existence of at least one further equilibrium. Note that the modification does not remove any other equilibriums, nor does it create a new equilibrium.
The restriction $\gamma_l < (1 - \psi^0 - c/(s^2/w\theta))/\phi^0$ ensures that the best responses to strictly positive symmetric effort levels close to zero are strictly positive for both types of workers. This is the basis to prove that the game is diagonally better reply secure in the sense of Reny (1999), although the diagonal payoff functions are not continuous at the point where both types of workers play an effort level of zero. Whenever there is a non-equilibrium point where all high-productivity workers play a symmetric effort level and where all low-productivity workers play a symmetric effort level, it is possible for at least one type of workers to reach a strictly higher

payoff than he would reach at any strategy profile in an open neighborhood of this strategy profile by unilaterally deviating to another effort level. Together with concavity of the utility functions in workers' own effort levels, which is provided by $\gamma_l < 0.5$, the existence of at least one point where the system (32)-(33) is fulfilled is ensured, provided the firm accepts any network applicant in the no-signal case.

## 5.4 Separating Equilibrium

### 5.4.1 Characterization

Consider the case when all workers apply the separating strategy described in case 2 of proposition 1 in phase 1. We will now characterize conditions such that this strategy might be played in equilibrium. First, we need to go back to the participation constraints for phase III as described in case 2 of Proposition 1. Next, we will analyze the workers' networking decisions in phase I and derive sufficient conditions for the existence of a separating equilibrium. Finally, we will analyze some properties of separating equilibria.
In the separating case, each high-productivity worker's probability to find a network referral can be obtained by substituting $e_l = 0$ and $e_{ip} = e_h$ into (8). The individual probability to find a network referral for a high productivity worker with arbitrary effort level $e_{ih}$ and the symmetric probability $\epsilon_h(e_h, 0)$ to find a network referral for all high productivity workers playing the symmetric effort level $e_h$ have convenient monotonicity properties.

**Lemma 9.** *(1) Suppose high-productivity worker ih plays $e_{ih}$, all other high-productivity workers play a symmetric effort level $e_h > 0$, and all low-productivity workers play $e_l = 0$. Then worker ih's probability to find a job via the network $\epsilon_{ih}(e_{ih}, e_h, 0)$ is (a) strictly increasing in $e_{ih} > 0$ and (b) strictly decreasing in $e_h$.*
*(2) If $e_{ih} = e_h$, the probability to find a job via the network $\epsilon_{ih}(e_h, e_h, 0) = e_h(e_h, 0)$ is strictly increasing in $e_h$.*

[Proof]

In a separating situation, increasing own effort always gives rise to a higher probability to find a job through the network, while a larger symmetric effort of the other high-productivity workers always results in a lower probability. A simultaneous increase in own effort and the effort of the other high-productivity workers has opposing effects. Since workers compete for jobs, but are also dependent on each other for network referrals, it is not obvious that the effect of the own effort prevails. However, in the current case, the larger probability to meet other workers who might give a referral outweighs the possibility that other workers might also be connected to a friend who has a referral.
As low-productivity workers do not exert any effort, their probability to find a network referral $\epsilon_{il}(0, e_h, 0)$ is equal to sorry. Thus, firms expect to encounter a high-productivity worker if they receive a network application without signal and therefore hire any worker who applies via the network. Thus, no vacancy matched to a jobless worker in the network will be posted on the formal labor market and no worker who finds a network referral will apply through the formal labor market. All other firms with vacancies and all other jobless workers will search through the formal labor market. Substituting into (9) we obtain the tightness of the formal labor market,

$$\vartheta(e_h, 0) = \frac{\theta - r\epsilon_h(e_h, 0)}{1 - r\epsilon_h(e_h, 0)} \tag{35}$$

**Lemma 10.** *Suppose workers apply the separating strategy in phase III and firms accept any application via the network.*
*(1) There are more vacancies than job-seekers on the formal labor market if and only if there*

*are more vacancies than job-seekers in the economy,*

$$\vartheta(e_h, 0) \geq 1 \Longleftrightarrow \theta \geq 1 \tag{36}$$

*(2) The labor market tightness of the formal labor market is higher than the labor market tightness in the whole economy if and only if there are more job-seekers than vacant jobs in the whole economy,*

$$\vartheta(e_h, 0) \geq \theta \Longleftrightarrow \theta \geq 1 \tag{37}$$

[Proof]

Each successful pairwise match in the network reduces the number of vacancies and the number of jobless workers by exactly 1. If $\theta > 1$, there are more vacant jobs than jobless workers in the economy. Hence, if there is a surplus/shortage of vacant jobs in the economy as a whole, there is still a surplus/shortage of vacant jobs after the network matching process. Thus, $\vartheta(e_h, 0) \geq 1$ if and only if $\theta > 1$. Furthermore, if there is a surplus of vacancies in the whole economy, the ratio of vacant jobs to jobless workers will be even larger after the network matching. By contrast, if there is a shortage of vacant jobs in the whole economy, the shortage will be even more striking after the network matching.

The following lemma provides another observation on $\vartheta(e_h, 0)$. It analyzes the effects of a change in $e_h$ on $\vartheta(e_h, 0)$.

**Lemma 11.** $\vartheta(e_h, 0)$ *is strictly increasing in* $e_h$ *for all* $e_h > 0$ *if and only if* $\theta \geq 1$. *Thus,*

$$\begin{aligned}
\vartheta(e_h, 0) &\in \left]\frac{\theta - r(1 - \exp[-\theta])}{1 - r(1 - \exp[-\theta])}, \theta\right[ \text{ if } \theta < 1 \\
\vartheta(e_h, 0) &= \theta = 1 \text{ if } \theta = 1 \\
\vartheta(e_h, 0) &\in \left]\theta, \frac{\theta - r(1 - \exp[-\theta])}{1 - r(1 - \exp[-\theta])}\right[ \text{ if } \theta > 1
\end{aligned}$$

[Proof]

Lemma 9 shows that the probability of each jobless high-productivity worker to find a match via the network $e_h(e_h, 0)$ is strictly increasing in $e_h$. Thus, the more effort the high-productivity workers invest in the network, the more high-productivity workers find a job through the network. The quantity of vacant jobs that are filled through the network equals the number of workers that find a job through the network. Hence, if originally there are more vacant jobs than job-seekers, the shortage of workers in the whole economy is aggravated by an increase in $e_h$. By contrast, if originally there are more job-seekers than vacant jobs, this shortage of vacant jobs becomes the larger, the more networking effort high-productivity workers invest in phase 1. $\vartheta(e_h, 0)$ is always between $\vartheta(0, 0) = \theta$ and

$$\lim_{e_h \to \infty} \vartheta(e_h, 0) = \frac{\theta - r(1 - \exp[-\theta])}{1 - r(1 - \exp[-\theta])}. \tag{38}$$

Thereby, if $\theta < 1$ ($\theta > 1$), $\vartheta(0, 0) = \theta$ is larger (smaller) than $\lim_{e_h \to \infty} \vartheta(e_h, 0)$.

The probability for high- and low-productivity workers to find a job through the formal labor market (conditional on entering it) in the separating equilibrium, $\psi(e_h, 0)$, can be obtained by substitution $\vartheta(e_h, 0)$ into (15). Consider now the decision of a firm that receives a network application. If the firm receives the signal $\gamma_h$, it will believe with probability 1 to encounter a high productivity applicant and therefore hire the applicant. On the equilibrium path, the firm will never receive signal $\gamma_l$. However, if the firm would receive signal $\gamma_l$ it would believe

with probability zero to encounter a high-productivity would and would not hire the network applicant. In the current separating case, if a firm does not receive any signal on the applicant's type, it expects to encounter a low-productivity worker with probability and thus hire him.
Consider a high-productivity worker's decision in phase III, after at least one other worker has offered him a referral. Anticipating that the firm will hire him, whenever he applies through the network, high-productivity workers will always choose the informal application channel.
On the equilibrium path, low-productivity workers will never face the decision whether to use the network application channel or the formal labor market, since they did not build a network. However, they might face this decision after a potential deviation from their zero-effort strategy in phase I. As we only consider unilateral deviations, in a large population, the respective deviation would not change the probability to find a job through the formal labor market, $\psi(e_h, 0)$. Consider now a low-productivity worker's decision after having received a referral offer by another worker in a deviation subgame. If he uses the referral offer and applies via the network, his type is revealed and the firm finds another worker on the formal labor market with probability $\gamma_l \phi(e_h, 0)$ and he stays unemployed. With probability $1 - \gamma_l \phi(e_h, 0)$ his type remains undisclosed and the firm hires him. He finds a job on the formal labor market with probability $\psi(e_h, 0)$. Corresponding to the general case in (19), the low-productivity worker uses the formal application channel in phase III if and only if

$$\gamma_l \geq \frac{1 - \psi(e_h, 0)}{\phi(e_h, 0)} = \overline{\gamma}(\vartheta(e_h, 0)) \tag{39}$$

This is the low-productivity workers' phase III separation participation constraint. Note that the low-productivity workers' decision whether to use the network in phase III is not part of any equilibrium path. Hence, there might be situations in which (39) is not fulfilled, but $BR_l(e_h, 0) = 0$ because the networking costs exceed the expected benefit from investing in social connections. From Lemma 2 we know that $\overline{\gamma}$ is in the interval $]0.5, 1[$.
We summarize the results regarding the threshold $\overline{\gamma}(\vartheta(e_h, 0))$ of Lemmata 2, 10 and 11 in Figure xxx (TBD!).
The solid (dashed) curve depicts the graph of $\overline{\gamma}(\theta)$ (of $\overline{\gamma}(\lim_{e_h \to \infty} \vartheta(e_h, 0))$). If $\theta \leq 1$, the family of functions $\overline{\gamma}(\vartheta(e_h, 0))$ for different $e_h$ (represented by the shaded area) is bounded from below by $\overline{\gamma}(\lim_{e_h \to \infty} \vartheta(e_h, 0))$ and bounded from above by $\overline{\gamma}(\theta)$. If $\theta > 1$, the function $\overline{\gamma}$ is bounded from above by $\overline{\gamma}(\lim_{e_h \to \infty} \vartheta(e_h, 0))$ and from below by $\overline{\gamma}(\theta)$. For each given $\theta$, $s$, $e_h$ and $r$, the actual threshold $\overline{\gamma}(\vartheta(e_h, 0))$ is in between these bounds. If the actual detection probability of low-productivty workers $\gamma_l$ is below the threshold $\overline{\gamma}(\vartheta(e_h, 0))$, then in phase III, low-productivity workers will use network referrals in case they receive a referral effort. If the actual detection probability $\gamma_l$ is above the threshold $\overline{\gamma}(\vartheta(e_h, 0))$, low-productivity workers will not use their potential referral offers in phase III, and consequently, they will not invest any networking effort in phase I. Since $\overline{\gamma}(\vartheta(e_h, 0))$ is never below 1/2, low-productivity workers always use the network in phase III if the probability that their type is not revealed is larger than the probability that it is disclosed. The only way to ensure the desired separating equilibrium in this case is through phase I, e.g. through a very high linking cost or low way. For any $e_h$, the threshold $\overline{\gamma}(\vartheta(e_h, 0))$ decreases in $\theta$. That is, the tighter the labor market, the more attractive is a referral offer in phase III, because also the formal labor market is the more competitive, the tighter the labor market in the economy as a whole. Therefore, low-productivity workers are willing to take a higher risk of being revealed and not employed if the labor market is tight. If $\theta < 1$, it is even for all $\gamma_l < 1/(\exp[1] - 1 \approx 0.582$ optimal to use any potential network referral offer in phase III, independent of $e_h$. By contrast, if $\theta > 1$, the formal labor market is more attractive for low-productivity workers. In that case, starting from the same threshold as above, if $\gamma_l > 1/(\exp[1] - 1 \approx 0.582$, no low-productivity worker will ever be willing to take the risk of

detection on the network. Hence, in this case, $BR_l(e_h, 0) = 0$ independently of $s$, $w$, $c$, and the high-productivity workers' networking effort.
These observations allow us to state conditions for the existence of a symmetric separating equilibrium: Firms accept any network applicant in phase IV and high-productivity workers accept any referral offered in phase III. Then it must be a best response for workers in phase I to play the symmetric effort level $e_h$ if they are of the high productivity type and they must play $e_l$ if they are of the low productivity type.
The first condition technically corresponds to the function $BR_{ih}(e_h, e_l)|_{e_l=0}$ having a fixed point respective to $e_h$, i.e. its graph intersecting the 45°-line at least once. The following lemma identifies a sufficient condition for this requirement.

**Lemma 12.** *If $c/(s^2/w) < \theta(1-\psi^0)$, then there exists at least one $e_h^* > 0$ such that $BR_h(e_h^*, 0) = e_h$.*

[Proof] The reasoning behind the proof is that the graph of the function $BR_{ih}(e_h, e_l)|_{e_l=0}$ eventually approaches $-\infty$ for $e_h \to \infty$. Hence, its slope will be negative starting from some effort level. Since its graph is continuous for all $e_h > 0$, it intersects the 45°-line at least once if $\lim_{e_h \to 0} BR_{ih}(e_h, 0) > 0$. The sufficient condition of Lemma 12 ensures that the graph of $BR_{ih}(e_h, 0)$ "starts" at a strictly positive level. Considering ratio $c/(s^2 w)$ we observe that a positive best response for high-productivity workers is more likely if the network costs $c$ are lower and the separating costs $s$ or the wage $w$ are higher.
The condition that low-productivity workers play zero effort in phase I can be fulfilled in several ways. One possibility is a sufficiently large $\gamma_l$ such that low-productivity workers do not accept any referral offer in phase III. In that case, $e_l = 0$ follows by sequential rationaly. Another possibility is that low-productivity workers would accept referral offers in phase III, but play zero-effort in phase I, e.g. if the networking effort cost $x$ is very high or $s$ and/or $w$ are low. We use the first of these possibilities to identify sufficient conditions for the existence of a separating equilibrium.

**Proposition 3.** *Let $c/(s^2)/w < \theta(1 - \psi^0)$. Then there exists a threshold $\overline{\gamma}^S$ such that $\gamma_l > \overline{\gamma}^S$ implies the existence of a separating equilibrium. In particular,*

- *if $\theta \geq 1$, then $\gamma_l > (1-\psi(\theta))/\phi(\theta)$ is sufficient for the existence of a separating equilibrium.*
- *if $\theta < 1$, then $\gamma_l > (1 - \psi(\lim_{e_h \to \infty} \vartheta(e_h, 0)))/\phi(\lim_{e_h \to \infty} \vartheta(e_h, 0)))$ is sufficient for the existence of a separating equilibrium.*

[Proof]

### 5.4.2 Analysis

Whenever a separating equilibrium exists, it co-exists with a zero-effort equilibrium. Therefore, we compare a labor market in which workers and firms coordinate on a separating equilibrium with a labor market in which workers and firms coordinate on the zero-effort equilibrium.

**Proposition 4.**

1. *The unemployment rate among righ-productivity workers is lower in any separating equilibrium than in the co-existing zero-effort equilibrium.*
2. *The unemployment rate among low-productivity workers is lower in any separating equilibrium than in the co-existing zero-effort equilibrium if and only if $\theta \geq 1$.*
3. *The probability that a filled vacancy is occupied by a worker with high productivity is larger in any separating equlibrium than in the co-existing zero-effort equilibrum.*

[Proof]

If there are more vacant jobs than jobless workers in the whole economy, $\theta \geq 1$, the surplus of vacancies relative to job-seekers is larger when the network has already remover a number of matches from the labor market (10). The formal labor market is the more promising for a worker, the larger the surplus of of vacant jobs relative to job-seekers (see 1). Hence, if $\theta \geq 1$, a separating situation results in a more promising formal labor market compared to the zero-effort situation. As the formal labor market is open to any worker, both high-productivity and low-productivity workers benefit from the higher labor market tightness.
If $\theta < 1$, there is a shortage of vacant jobs in the economy and the network matches render the formal labor market more competitive than the initial labor market. From a worker's perspective, the formal labor market is less promising in the separating situation than in a zero-effort situation. As low-productivity workers can only apply on the formal labor market, they are worse-off in the separating situation than in a zero-effort situation. For high-productivity workers, the effect on their probability to find a job on the formal labor market is also detrimental. However, in the separating situation, they only use the formal labor market if they did not find a job via the network. We show in Proposition 4 that the probability $\rho(e_h, 0)$ to find a job in a separating equilibrium on either market is larger than the probability $\psi^0$ to find a job in the formal labor market in a zero-effort situation. Consider the limit of $\rho(e_h, 0)$ if $e_h$ approaches zero. The probability to find a job via the network approaches zero, while the probability to find a job via the formal labor market approaches $\psi^0$. Hence, if $e_h$ is close to zero, the probability to find a job in a separating equilibrium is very close to the probability to find a job in a zero-effort situation. The key to the proof of Proposition 4 is to find a function $\tilde{\rho}(e_h, 0)$ which bounds the family of functions $\rho(e_h, 0)$ for different values of $r$ from below. For instance, the limit of $\rho(e_h, 0)$ if $r \to 1$, i.e. the probability to find a job in a situation without low-productivity workers, could be used to create such a function $\tilde{\rho}$. We show that $\tilde{\rho}(e_h, 0)$ is increasing in $e_h$ despite the following two opposing effects. On the one hand, the probability to find a job via the network, $\epsilon_h(e_h, 0)$ increases in $e_h$ (see Lemma 9). On the other hand, the probability to find a job through the formal labor market decreases in $e_h$, because other high-productivity workers who are matched to vacant jobs via the network reduce the labor market tightness for the remaining high-productivity workers. We show that the direct effect on $\epsilon_h(e_h, 0)$ (an increased $e_{ih}$ induces a larger probability to find a job via the network even if all other high-productivity workers also invest more effort) compensates the rather indirect effect on $\psi(e_h, 0)$ (a larger networking effort of the other high-productivity workers reduces the available number of jobs per job-seeker and therefore decreases the probability to find a job in the formal labor market if no job was found via the network). This implies that $\tilde{\rho}(e_h, 0) > \psi^0$. Since $\rho(e_h, 0) > \overline{\rho}(e_h, 0) \forall 0 < r < 1$, it follows that $\rho(e_h, 0) > \psi^0$.

More discussion!

### 5.5 Pooling Equilibrium

We will now analyze in more detail potential pooling equilibria, i.e. equilibria in which both types of workers play a strictly positive effort level in phase I. We proceed by backward induction, starting with the firms' decision in phase IV.

#### Phase IV

Suppose that workers have played a symmetric strategy profile in phase I and a pooling strategy in phase III, i.e. all high and low productivity workers who have had a network referral have used it. As discussed in section 5.1 In any possible equilibrium, on the equilibrium path, firms will

always hire their respective network applicant directly if they receive the high-productivity signal and never hire the respective applicant directly if they receive the low-productivity signal.
Consider a firm that interviews a network application and does not receive any productivity signal. The decision whether or not to hire the applicant depends on the firm's beliefs regarding the proportions of high- and low-productivity workers in the network and on the formal labor market. For any worker, the probability that another worker is of the high-productivity type, conditional on having applied at a firm via the referral of worker $j$ and on the firm not receiving any signal can be obtained using Bayes' rule as[5]

$$\frac{rg_{jh}\alpha_h(1-\gamma_h)}{rg_{jh}(1-\gamma_h)\alpha_h+(1-r)g_{jl}(1-\gamma_l)\alpha_l}. \tag{40}$$

On the formal labor market, there are $M_{Ah} = rs(1-\epsilon_h(e_h,e_l))M$ high-productivity workers and $M_{Al} = (1-r)s(1-\epsilon_l(e_h,e_l))M$ low-productivity workers. Therefore, the probability that a randomly chosen worker on the formal labor market is highly productive is

$$\frac{r(1-\epsilon_h)}{r(1-\epsilon_h)+(1-r)(1-\epsilon_l)}. \tag{41}$$

A firm prefers the network applicant if the probability that this worker is highly productive is lager than the probability of a random worker from the formal labor market. These probabilities depend on the signals $\gamma_l$ and $\gamma_h$. After some transformations we obtain a sufficient and necessary condition for the firm to prefer the network applicant:

$$\frac{rg_{jh}\alpha_h(1-\gamma_h)}{rg_{jh}(1-\gamma_h)\alpha_h+(1-r)g_{jl}(1-\gamma_l)} > \frac{r(1-\epsilon_h)}{r(1-\epsilon_h)+(1-r)(1-\epsilon_l)} \tag{42}$$

$$\Leftrightarrow \frac{1-\gamma_l}{1-\gamma_h} < \frac{g_{jh}\alpha_h(1-\epsilon_l)}{g_{jl}\alpha_l(1-\epsilon_h)} \tag{43}$$

We will refer to (43) as the firm's participation constraint in a pooling equilibrium.

**Lemma 13.** *If $\gamma_l \geq \gamma_h$ and all players play $e = (e_h, e_l)$ with $e_h > e_l$, then (43) is fulfilled.*

[Proof]
Note that, as we will prove below, $e_h > e_l$ in any symmetric equilibrium. Hence, if $\gamma_l \geq \gamma_h$, any symmetric strategy profile in which all workers play a best response given by equations (29) and (31). For $\gamma_l > \gamma_h$, it is optimal for firms to employ whose type is not revealed because these are more likely to be highly productive. In contrast, for $\gamma_l < \gamma_h$, the firm's participation constraint is not automatically fulfilled.

### Phase III

Whenever the firm prefers network applicants over formal applicants in case it does not receive any signal in phase III, high productivity workers always prefer to use the network channel if they have the opportunity to do so. Low productivity workers however, encounter the risk that their type is revealed through the network. In this case, they will only be hired if the firm does not find a match on the formal labor market. Recall that, according to (19), for any fixed $e_h > 0$ and $e_l > 0$, low-productivity workers accept referral offers in phase III if the probability of the low-productivity signal is below the threshold $\overline{\gamma}(\vartheta(e_h, e_l)) \equiv (1 - \psi(e_h, e_l)/\phi(e_h, e_l))$.

**Lemma 14.** *If $\gamma_l < 0.5$, low-productivity workers always accept any referral offer in phase III.*

[5] The probability to be high-productive is $r$. For type $p$, the probability to apply to the firm via a referral of worker $j$ is $g_{jp}\alpha_p$ (see (5)). The probability that there is no signal for applicant type $p$ is $1-\gamma_p$.

Lemma 14 follows directly from Lemma 2

**Phase I**

We compare the best response function of high- and low-productivity workers (29) and (31) and deduct the following observation for $e_h > 0$ and $e_l > 0$, given that low productivity workers' phase III participation constraint in a pooling equilibrium (19) is fulfilled.

**Lemma 15.** *For any given $(e_h, e_l)$ with $e_h > 0$ and $e_l > 0$, a high-productivity worker's phase I best response is larger than a low-productivity worker's phase I best response.*

[Proof]

This allows us to make an interesting observation about equilibrium effort levels. Since workers play a best response to the strategy profile of the other workers, any equilibrium effort level of the high-productivity workers will be larger than the corresponding effort level of the low-productivity workers.

**Proposition 5.**

1. *The probability that a filled vacancy is occupied by a high-productivity worker is larger in any pooling equilibrium than in the co-existing zero-effort equilibrium.*
2. *The overall employment rate and the unemployment rate among high-productivity workers are lower in any pooling equilibrium than in the co-existing zero-effort equilibrium.*
3. *If $\theta > 1$, both the unemployment rate among high-productivity workers and among low-productivity workers are lower in any pooling equilibrium than in the co-existing zero-effort equilibrium.*

[Proof]

Firms always benefit from a transition from the zero-effort equilibrium to any pooling equilibrium. They not only benefit from a better match quality but also from a more efficient matching mechanism.

The effect on the match quality is ambiguous upfront. On the one hand, the network works as screening device. Even if the applicant's type is not revealed, the proportion of high productivity workers among workers with a network referral is larger than in the whole population because in equilibrium, high productivity workers invest more effort in networking than low-productivity workers. On the other hand, the proportion of low-productivity workers on the formal labor market is higher than the original proportion and therefore the probability to hire a low-productivity worker is also higher if the vacancy is filled via the formal labor market. However, as the total probability to find a job is higher for a high productivity worker, the proportion of high-productivity workers among those originally unemployed workers who find a new job must always be larger in the pooling equilibrium than in an equilibrium where both types of workers are matched with the same probability.

Furthermore, Proposition 5 describes an effect of the use of the network on the efficiency of the matching process. In a pooling equilibrium, there are weaker search frictions than in a co-existing equilibrium in which workers do not form a network. Since precisely one firm and one worker are involved in a match, the total number of unfilled jobs and unemployed workers is lower in the pooling equilibrium than in the zero-effort equilibrium.

Consider first the case when $\vartheta(e_h^*, e_l^*) \leq \theta$, i.e., in the pooling equilibrium, the labor market tightness on the formal labor market is lower than or equal to the labor market tightness in the whole economy. Denote by $\eta_i(e_h, e_l)$ the probability that an arbitrary vacancy $i$ is filled via the network. Then, the total probability that this vacancy is filled at the end of the game is equal to

$$\eta_i(e_h^*, e_l^*) + (1 - \eta_i(e_h^*, e_l^*))\phi(e_h^*, e_l^*) = \phi(e_h^*, e_l^*) + (1 - \phi(e_h^*, e_l^*))\eta_i(e_h^*, e_l^*) \tag{44}$$

For $\eta_i(e_h^*, e_l^*) > 0$, this expression is strictly larger than $\phi(e_h^*, e_l^*)$, that is, the probability that a vacancy is filled through the formal labor market in the pooling equilibrium $(e_h^*, e_l^*)$. As firms benefit on the formal labor market from a decreased tightness $\vartheta$ (see Lemma 1), $\phi(e_h^*, e_l^*)$ is larger than $\phi^0$, the probability that the vacancy is filled in the zero-effort equilibrium.

Next, consider $\vartheta(e_h^*, e_l^*) > \theta$, i.e. in the pooling equilibrium, the ratio of vacancies and workers searching on the formal labor market is larger than in the whole economy. Then the probability $\rho_p(e_h, e_l)$ that an unemployed worker of type $p$ finds a job in a pooling equilibrium with $(e_h^*, e_l^*)$ is larger than $\psi(e_h^*, e_l^*)$, i.e. the probability to find a job on the formal labor market in that pooling equilibrium (see (24) and (25) and recall that in equilibrium, (19) holds). Since job-seekers benefit from an increase in the labor market tightness $\vartheta$ (see Lemma 1), in the pooling equilibrium, the probability that an unemployed worker finds a job on the formal labor market is higher than the total probability to find a job in the zero-effort equilibrium $\psi^0$. Thus, the expected number of workers of both types finding a job in the pooling equilibrium is higher than in the zero-effort equilibrium and therefore also the number of filled vacancies.

The effect of a transition from a zero-effort equilibrium to a pooling equilibrium on workers is mixed. If $\theta$ is large, $\vartheta(e_h^*, e_l^*) > \theta$ and the probability to find a job on the formal labor market in the pooling equilibrium is larger than the total probability to find a job in the zero-effort equilibrium and therefore, also the total probability to find a job in the pooling equilibrium. Thus, both types of workers have a lower probability to stay unemployed in the pooling equilibrium than in the zero-effort equilibrium.

If $\theta$ is low, high productivity workers benefit from network formation while low-productivity workers suffer from a higher probability to stay unemployed. In that case, $\vartheta(e_h^*, e_l^*)$ and the probability to find a job on the formal labor market in the pooling equilibrium is lower than the total probability to find a job in the zero-effort equilibrium. For workers with high productivity this detrimental effect is offset by the beneficial effect of the opportunity to use the network and the network's screening function. The firms' equilibrium beliefs are such that they hire any network applicant in absence of a signal. Hence, workers with high productivity who find a network referral will always be hired. Furthermore, as high-productivity workers invest more in the network than low-productivity workers, their probability to obtain a network referral is larger than that of low-productivity workers. For low-productivity workers, the detrimental effect prevails despite the opportunity to use the network. On the one hand, on average, proportion $\gamma_l$ of those workers who have a network referral will not find any job because their type is revealed to the firm. On the other hand, the proportion of low-productivity workers who have a network referral is lower than that of high-productivity workers because low-productivity workers invest less networking effort in equilibrium.

# 6 Welfare

Our results suggest that equilibria with networking might be favorable over the co-existing zero-effort equilibrium, since overall unemployment and unemployment among-high-productivity workers are lower and the average match quality is higher in the former type of equilibria. However, this consideration does ignore the individual and social costs of networking. The individual payoff functions 26 are a reasonable measure for individual welfare. Social welfare $W$ is the surplus of the aggregate production over the socialization cost. In any symmetric equilibrium $(e_h^*, e_l^*)$, $W$ can be expressed as function of $e_h^*$ and $e_l^*$:

$$W(e_h^*, e_l^*) = M[r(1 - s + s\rho_h(e_h^*, e_l^*))p_h + (1 - r)(1 - s + s\rho_l(e_h^*, e_l^*))p_l - c(re_h^* + (1 - r)e_l^*)] \tag{45}$$

**Proposition 6.** *The payoffs of both types of workers and total welfare are larger in any separating equilibrium than in the co-existing zero-effort equilibrium whenever* $\theta > 1$. *In any pooling equilibrium, these values are larger than in the co-existing zero-effort equilibrium whenever* $\theta > \vartheta(e_h^*, e_l^*)$.

*Proof.* Let $(e_h^*, e_l^*)$ be an arbitrary, symmetric non-zero effort equilibrium. As $e_h^*$ and $e_l^*$ are best-responses,

$$sw\rho_h(e_h^*, e_l^*) - ce_h^* \quad > \quad sw\psi(e_h^*, e_l^*) \text{ and} \tag{46}$$
$$sw\rho_l(e_h^*, e_l^*) - ce_l^* \quad > \quad sw\psi(e_h^*, e_l^*). \tag{47}$$

Hence,

$$e_h^* < sw/c[1 - \psi(e_h^*, e_l^*)]\epsilon_h(e_h^*, e_l^*) \text{ and} \tag{48}$$
$$e_l^* < sw/c[1 - \psi(e_h^*, e_l^*) - \gamma_l \phi(e_h^*, e_l^*)]\epsilon_l(e_h^*, e_l^*) \tag{49}$$

It follows,

1. $\pi_h(e_h^*, e_l^*) - \pi_h(0,0) = sw[\rho_h(e_h^*, e_l^*) - \psi^0] - ce_h^* \geq sw[\rho_h(e_h^* - e_l^*) - \psi^0] - c[sw/c(1 - \psi(e_h^*, e_l^*))]\epsilon_h(e_h^*, e_l^*) = sw(\psi(e_h^*, e_l^*) - \psi^0)$.
2. $\pi_l(e_h^*, e_l^*) - \pi_l(0,0) = sw[\rho_l(e_h^*, e_l^*) - \psi^0] - ce_l^* \geq sw[\rho_l(e_h^* - e_l^*) - \psi^0] - c[sw/c(1 - \psi(e_h^*, e_l^*) - \gamma_l \phi(e_h^*, e_l^*))]\epsilon_l(e_h^*, e_l^*) = sw(\psi(e_h^*, e_l^*) - \psi^0)$.

Using $p_h > p_l > w$ (CHECK!!), this implies $W(e_h^*, e_l^*) - W(0,0) > r\pi_h(e_h, e_l^*) + (1-r)\pi_l(e_h^*, e_l^*) - r\pi_h(0,0) - (1-r)\pi_l(0,0) = sw[\psi(e_h^*, e_l^*) - \psi^0]$. Hence, whenever $\psi(e_h^*, e_l^*) - \psi \geq 0 \iff \vartheta > \theta$, it follows that $\pi_h(e_h^*, e_l^*) > \pi_h(0,0), \pi_l(e_h^* e_l^*) > \pi_l(0,0)$, and $W(e_h^*, e_l^*) > W(0,0)$. In case of separating equilibria, $\vartheta(e_h^*, e_l^*) \geq \theta \iff \theta \geq 1$. ■

Hence, if $\theta$ is high, i.e. if in the non-zero effort equilibrium $\vartheta(e_h^*, e_l^*) \geq \theta$, ...
In the opposing case, i.e. if in the non-zero effort equilibrium $\vartheta(e_h^*, e_l^*) < \theta$ (CHECK!), individual welfare for a worker of type $l$ is lower in the non-zero effort equilibrium, since by propositions 4 and 5 the probability to find a job if unemployment $\rho_p(e_h^*, e_l^*)$ is lower in the non-zero effort equilibrium than in the co-existing zero effort equilibrium. On the other hand, high productivity workers might have a higher or a lower pooling or separating payoff in these cases since they have an increase probability to find a job but invest costly effort in networking. Additionally, whether social welfare in the respective non-zero effort equilibria is higher or lower than in the co-existing zero-effort equilibria also depends on the productivity of high- and low-productivity workers, $p_h$ and $p_l$. As $e_h^*$ and $e_l^*$ are independent of $p_h$ and $p_l$, social welfare might be larger than in a zero effort equilibrium in the case of a low difference of $p_h$ and $p_l$ and lower in case of a smaller difference.

# 7 Conclusion

TBD

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# Appendix

### Proof of Lemma 1

*Proof.* The first and second derivatives of $\psi(\theta)$ are

$$\frac{\partial\psi(\vartheta)}{\partial\vartheta} = 1 - \frac{1+\vartheta}{\vartheta[\exp[1/\vartheta]]} \tag{50}$$

$$\frac{\partial^2\psi(\vartheta)}{\partial\vartheta^2} = \frac{\exp[-1/\vartheta]}{\vartheta^{-3}} < 0 \tag{51}$$

From

$$\lim_{\vartheta\to\infty}\left(1 - \frac{1+\vartheta}{\vartheta\exp[1/\vartheta]}\right) = 0 \tag{52}$$

it follows that $\partial\psi(\vartheta)/\partial\vartheta > 0 \;\forall\; 0 < \vartheta < \infty$.

The first derivative of $\phi(\vartheta)$ is

$$\frac{\partial\phi(\vartheta)}{\partial\vartheta} = -\frac{\exp[-1/\vartheta]}{\vartheta^2} < 0 \tag{53}$$

■

### Proof of Lemma 2

*Proof.* In step 1, we show that $\overline{\gamma}(\vartheta)$ is decreasing in $\vartheta \;\forall\; \vartheta > 0$. Then, in step 2, we show that 1/2 is a lower bound to $\overline{\gamma}(\vartheta)$. In step 3, we provide a proof for 1 as an upper bound to $\overline{\gamma}(\vartheta)$. Finally, in step 4, we show the ranges of $\overline{\gamma}(\vartheta)$ if $\vartheta = 1$, $\vartheta > 1$ and $\vartheta < 1$.

1. The first derivative of $\overline{\gamma}(\vartheta)$ is

$$\frac{\partial\overline{\gamma}(\vartheta)}{\partial\vartheta} = -1 - \left(\frac{\partial\phi(\vartheta)}{\partial\vartheta}\phi(\vartheta)^{-2}\right) = -1 + \frac{(1-\phi(\vartheta))}{\vartheta^2\phi(\vartheta)^2} \tag{54}$$

Thus,

$$\frac{\partial\overline{\gamma}(\vartheta)}{\partial\vartheta} < 0 \tag{55}$$

$$\Longleftrightarrow 1 - \phi(\vartheta) < \psi(\vartheta)^2 \tag{56}$$

$$\Longleftrightarrow \sqrt{1-\phi(\vartheta)} < \psi(\vartheta) \Longleftrightarrow \sqrt{\exp[-1/\vartheta]} < \vartheta(1-\exp[-1/\vartheta]) \Longleftrightarrow \tag{57}$$

$$\frac{\sqrt{\exp[1/\vartheta]}}{\exp[1/\vartheta]-1} < \vartheta \tag{58}$$

Now substitute $\exp[1/\vartheta]$ in (58) by $a$. Then, (58) $\Longleftrightarrow$

$$\frac{\sqrt{1}}{a-1} < \vartheta = \frac{1}{1/\vartheta} = \frac{1}{\ln[\exp[1/\vartheta]]} = \frac{1}{\ln[a]} \Longleftrightarrow \tag{59}$$

$$a - 1 > \ln[a]\sqrt{a} \tag{60}$$

Since $\vartheta > 0$, if follows that $\exp[1/\vartheta] = a > 1$. The LHS of (60) is linearly increasing in $a$ and the RHS is increasing and concave in $a > 1$. If $a \to 1$, both sides approach zero. Since the first derivative of both sides approaches the same value, i.e. 1 if $a \to 1$, we can conclude that the first derivative of the RHS is strictly smaller than 1 whenever $a > 1$. Thus, for all $a > 1$, the function value of the RHS is strictly smaller than the value on the LHS. Thus, (60) is fulfilled for all $a > 1$ and (56) holds for all $\vartheta > 0$.

2. Since $\overline{\gamma}(\vartheta)$ is decreasing in $\vartheta$, and since for large $\theta$, $\vartheta > \theta$, $\overline{\gamma}(\vartheta) > \overline{\gamma}(\theta) > \lim_{\theta\to\infty} \overline{\gamma}(\vartheta) = 1/2$. The limit is obtained by applying L'hôspital's rule.

$$\lim_{\theta\to\infty} \frac{1-\psi\theta}{\phi(\theta)} = \lim_{\theta\to\infty} \frac{\partial[1-\psi(\theta)]/\partial\theta}{\partial\phi(\theta)/\partial\theta} = \lim_{\theta\to\infty} \frac{-1-\exp[-1/\theta](1+\theta^{-}1)}{-\exp[-1/\theta]\theta^{-2}} \tag{61}$$

$$= \lim_{\theta\to\infty} \frac{\exp[-1/\theta]\theta^{-3}}{\exp[-1/\theta](2\theta-1)\theta^{-4}} = \lim_{\theta\to\infty} \frac{\theta}{2\theta-1} = \frac{1}{2} \tag{62}$$

3. For $\vartheta$ close to zero, $\vartheta \leq \theta$. Hence, $\overline{\gamma}(\vartheta) < \overline{\gamma}(\theta) < \lim_{\theta\to 0} \overline{\gamma}(\vartheta) = 0$. The limit follows from $\lim_{\theta\to 0}[1-\psi(\theta)] = 1-\theta[1-\exp[-1/\theta]] = 1-0\cdot 1 = 1$ and $\lim_{\theta\to 0}\phi(\theta) = 1-\exp[-1/\theta] = 1-0 = 1$.

4. If $\vartheta = 1$, then $\overline{\gamma}(1) = [1-\psi(1)]/\phi(1) = [\exp[1]-1]^{-1} \approx 0.5819$.
If $\vartheta > 1$, then $\overline{\gamma}(\vartheta) < [\exp[1]-1]^{-1}$ since $\overline{\gamma}(\vartheta)$ is decreasing in $\vartheta$.
Furthermore, $\overline{\gamma}(\vartheta) > \overline{\gamma}(\theta) > \lim_{\theta\to\infty} \overline{\gamma}(\theta) = 0.5$.
If $\vartheta < 1$, then $\overline{\gamma}(\vartheta) > [\exp[1]-1]^{-1}$ since $\overline{\gamma}(\vartheta)$ is decreasing in $\vartheta$.
Furthermore, $\overline{\gamma}(\vartheta) < \overline{\gamma}(\theta) < \lim_{\theta\to\infty} \overline{\gamma}(\vartheta) = 1$.

■

**Proof of Lemma 3**

*Proof.* Since $\psi(\vartheta) < 1$ and $\phi(\vartheta) < 1$, inequality (56) is a sufficient condition for (20) to be fulfilled. The proof for (56) is given in the proof of Lemma 2, part 1. ■

**Proof of Lemma 4**

*Proof.* 1. **"If"-Part:** This part of the proof is by contradiction. Consider a worker for whom it is optimal to choose a strategy in phase I such that $e_{ip} > 0 \; \forall \; i \in \mathcal{M}$. for at least one type $p \in \{h, l\}$ and suppose it is optimal for him in case he is of type $p \in \{h, l\}$ to search on the formal labor market in phase III, despite having a referral offer. As socialization is costly and does not yield any benefits for type $p$, it is optimal to set $e_{ip} = 0$. This contradicts $e_{ip} > 0$.

2. **"Only If"-Part:** Suppose a worker is of type $o$ with $e_{ip} = 0$, i.e. the only remaining type for which $e_{ip}$ is not strictly positive. Then, on the equilibrium path, he cannot receive a network effort and thus cannot use the network.

■

**Proof of Proposition 1**

*Proof.* We will show that situations that do not belong to one of the cases 1 to 3 cannot be a symmetric pure strategy PBE.
A symmetric phase I pure strategy profile of workers is such that $e_i(e_h, e_l) \; \forall \; i \in \mathcal{M}$, with $e_h \geq 0$. Hence, there are four cases of symmetric phase I pure strategy profiles of workers: (1) $e_i = (0, 0)$ $\forall \; i \in \mathcal{M}$, (2) $e_i(e_h, 0) \; \forall \; i \in \mathcal{M}$ with $e_h > 0$, (3) $e_i = (e_h, e_l) \; \forall \; i \in \mathcal{M}$ with $e_h > 0$ and $e_l > 0$,

and (4) $e_i = (0, e_l)\ \forall\ i \in \mathcal{M}$ with $e_l > 0$.
In case (1), workers do not face a decision in phase III. In cases (2) – (4), a symmetric phase III action profile of workers is such that all workers of the same type either use the network if they have that possibility or they do not use it. By Corollary 1, on any equilibrium path, all workers who have a network referral will use it. Finally, we show that (4) cannot be played in a weak PBE by contradiction. Suppose $e_i = (0, e_l)\ \forall\ i \in \mathcal{M}$. Then, only low-productivity workers can potentially receive a network referral. By weak consistency, firms infer that any potential network applicant is of the low-productivity type with probability 1, whereas an arbitrary applicant on the formal labor market is of the high-productivity type with a strictly positive probability. Hence, firms do not hire network applicants directly. By Lemma 3, low-productivity workers who expect firms not to hire their network applicants directly in the no-signal case do not use their potential network referrals in phase III. By sequential rationality, they do not invest any effort in phase I in that case. This contradicts $e_l > 0$. ∎

**Proof of Lemma 5**

*Proof.* In order to prove that the numerator of (21) is strictly positive, we will show that $\epsilon_p(e_h, e_l) < \theta\ \forall\ \theta,\ e_h,\ e_l \geq 0$ and for all $p \in \{h, l\}$.
Denote by $p$ the worker type such that $e_p = \max(e_h, e_l)$. Denote by $q$ the worker type such that $e_q = \min(e_h, e_l)$. Denote

$$f^t(e_t) = \begin{cases} f(e_h, 0), & \text{if } t = h \\ f(0, e_l), & \text{if } t = l \end{cases} \quad \text{and } r_t = \begin{cases} r, & \text{if } t = h \\ 1 - r, & \text{if } t = l \end{cases} \tag{63}$$

for all $t \in \{p, q\}$:
We use two intermediary steps (steps 1 and 2) to prove that $\theta > \epsilon_p(e_h, e_l)\ \forall\ p \in \{h, l\}$ in step 3.

1. First, we show that $f^q(e_q) \geq f(e_h, e_l) \geq f^p(e_p)$.
   To see this, consider the following transformation: $f(e_h, e_l) = [re_h + (1-r)e_l]^{-1}\theta[-1 + r\exp[-e_h s] + (1-r)\exp[-e_l s]] = [re_h + (1-r)e_l]^{-1}[re_h f(e_h, 0) + (1-r)e_l f(0, e_l)] = (r_p e_p + r_q e_q)^{-1}[e_p e_p f^p(e_p) + r_q e_q f^e(e_q)]$. As $e_t f^t(e_t)$ is strictly decreasing in $e_t$ for all $t \in \{p, q\}$ and $0 < r_p, r_q < 1$ and $r_p + r_q = 1$, it follows that $(r_p e_p + r_q e_q)^{-1} e_q f^e(e_q) \geq f(e_h, e_l) \geq (r_p e_p + r_q e_q)^{-1} e_p f^p(e_p)$. As $\frac{e_p}{r_p e_p + r_q e_q} \geq 1 \geq \frac{e_q}{r_p e_p + r_q e_q}$, it follows that $f^q(e_q) \geq f(e_h, e_l) \geq f^p(e_p)$.

2. Second, we show that $\epsilon(e_h, e_l) < 1 - \exp[-\theta]$.
   Note that $\epsilon_p$ is decreasing in $f(e_h, e_l)$. Hence, $\epsilon_p(e_h, e_l) = 1 - \exp[e_p f(e_h, e_l)] \leq 1 - \exp[e_p f^p(e_p)]$. The latter expression is strictly increasing in $e_p$. As $\lim_{e_p \to \infty} 1 - \exp[e_p f^p(e_p)] = 1 - \exp[-\theta]$, it follows that $\epsilon_p(e_h, e_l) \leq 1 - \exp[-\theta]$.

3. Last, we show that $\theta > 1 - \exp[-\theta]\ \forall\ \theta > 0$ by comparing the two expressions $\theta$ and $1 - \exp[-\theta]$.
   Observe that (1) $\lim_{\theta \to 0} \theta = \lim_{\theta \to 0}(1 - \exp[-\theta]) = 0$, (2)$\lim_{\theta \to 0}(\partial\theta/\partial\theta) = \lim_{\theta \to 0} \partial[1 - \exp[-\theta]]/\partial\theta = \lim_{\theta \to 0} \exp[-\theta])1$, and (3) $\theta$ is linearly increasing $\in \theta$ for all $\theta > 0$ while $1 - \exp[\theta]$ is concavely increasing in $\theta$ for all $\theta > 0$. Hence, $\theta > 1 - \exp[-\theta]\ \forall\ \theta > 0$.

As $r < 1$ and $\gamma_l < 1$, the latter inequality implies that the numerator of (21), i.e. $r(\theta - \epsilon_h(e_h, e_l)) + (1-r)(\theta - (1-\gamma_l)\epsilon_l(e_h, e_l))$ is strictly positive for all $e_h,\ e_l,\ \theta > 0$. ∎

**Proof of Lemma 6**

*Proof.* Suppose that all workers $j \in \mathcal{M}\backslash\{i\}, j \neq i$ play $e_i = (0, 0)$. Consider worker $ip$ of type $p \in \{h, l\}$. $\epsilon_{ip}(e_{ip}, 0, 0) = 0\ \forall\ e_{ip} \geq 0$ and for all $p \in \{h, l\}$, his payoff is $-ce_{ip}$, regardless of the play off the equilibrium path. This is maximized by playing $e_{ip} = 0$. Hence, player $i$'s best response is zero if he is of the high, and zero if he is of the low-productivity type. Therefore, a phase I strategy profile where $e_{ip} = (0, 0)\ \forall\ i \in \mathcal{M}$ and $\forall\ p \in \{h, l\}$ is always part of a PBE. ■

### Proof of Lemma 7

*Proof.* The proof for the unemployment rates follows directly from the definitions. As each worker loses his job with the same probability and finds a new job with the same probability $\psi^0$, regardless of his type, the average probability for a vacancy to be filled by a highly productive worker is equal to the proportion of highly productive workers in the population, i.e., $r$. ■

### Proof of Lemma 8

*Proof.*

$$\frac{\partial^2 \Pi_{ip}(e_{ip}, e_h, e_l)}{\partial e_{ip}^2} = sw \frac{\partial^2 \rho_{ip}(e_{ip}, e_h, e_l)}{\partial e_{ip}^2} \tag{64}$$

As $sw$ is strictly positive, the assertion of this lemma is equivalent to the following claim: For all $p \in \{h, l\}$, $\rho_{ip}(e_{ip}, e_h, e_l)$ is strictly concave in $e_{ip}$, for $e_h > 0$ or $e_l > 0$ if $1 - \psi(e_h, e_l) - \gamma_l \phi(e_h, e_l) > 0$. If $1 - \psi(e_h, e_l) - \gamma_l \phi(e_h, e_l) < 0$, then $\rho_{ih}(e_{ih}, e_h, e_l)$ is strictly concave in $e_{ih}$ and $\rho_{il}(e_{il}, e_h, e_l)$ is strictly convex in $e_{il}$.
We first show that for both types of workers the latter formulation is equivalent to the claim that $\epsilon_{ip}(e_{ip}, e_h, e_l)$ is strictly concave in $e_{ip}$ for $e_h > 0$ or $e_l > 0$ (step 1). In step 2 we prove this claim.

1. Rewrite $\rho_{ih}(e_{ih}, e_h, e_l) = \epsilon ih(e_{ih}, e_h, e_l)(1 - \psi(e_h, e_l)) + \psi(e_h, e_l)$ and $\rho_{il}(e_{il}, e_h, e_l) = \epsilon il(e_{il}, e_h, e_l)(1 - \psi(e_h, e_l) - \gamma_l \phi(e_h, e_l)) + \psi(e_h, e_l)$.
   Denote

$$h_p(e_h, e_l) = \begin{cases} 1 - \psi(e_h, e_l) & \text{if } p = h \\ 1 - \psi(e_h, e_l) - \gamma_l \phi(e_h, e_l) & \text{if } p = l \end{cases} \tag{65}$$

   As $\psi(e_h, e_l)$ is independent of $e_{ip}$, $[\partial^2 \rho(e_{ip}, e_h, e_h)]/(\partial e_{ip}^2) = [\partial^2 \epsilon_{ip}^2(e_{ip}, e_h, e_h)]/(\partial e_{ip}^2) h_p(e_h, e_l)$. Hence, if $h_p(e_h, e_l) \geq 0$, the sign of $[\partial^2 \rho(e_{ip}, e_h, e_h)]/(\partial e_{ip}^2)$ is equal to the sign of $\epsilon_{ip}(e_{ip}, e_h, e_l)$, while if $h_p(e_h, e_l) < 0$, it is the sign of $-\epsilon_{ip}(e_{ip}, e_h, e_l)$.

2. 

$$\frac{\partial \epsilon_{ip}(e_{ip}, e_h, e_l)}{\partial e_{ip}} = -\exp[e_{ip} f(e_h, e_l)] f(e_h, e_l)^2 < 0 \quad \forall \quad e_h > 0 \text{ or } e_l > 0. \tag{66}$$

■

### Proof of Proposition 2

We will prove the existence result of Proposition 2 using a well-known theorem by Reny (1999) (Theorem 4.1). This theorem states four sufficient conditions for the existence of a symmetric equilibrium in a discontinuous game: (1) compactness, (2) quasi-symmetry, (3) diagonal quasi-concavity, and (4) diagonal better reply security. We first recall the respective definitions of these properties.

**Definition 1.** **A compact game** (see Reny (1999), p. 1032) is defined as a game in which each player's set of strategies is a nonempty, compact subset of a topological vector space, and all payoff functions are bounded.

**Definition 2.** (see Reny (1999), p. 1040) Let $\Gamma = (X_1, \pi_i)_{i=1}^N$ be an N-player game with strategy space $\mathbf{X}$ such that $\mathbf{X} = X_1 \times ... \times X_N$ and $X_1 = ... = X_N = X$. The game is **quasi-symmetric** if for all players $\pi_1(x, y, ..., y) = \pi_2(y, x, y, ..., y) = ... = \pi_N(y, ..., y, x)$.

**Definition 3.** (see Reny (1999), p. 1040) A game $\Gamma = (X_1, \pi_i)_{i=1}^N$ is **diagonally quasi concave** if X is convex and for every player $i$ and all $x^1, ..., x^m \in X$, and all $\overline{x} \in co(x^1, ..., x^m)$

$$\pi_i(\overline{x}, ..., \overline{x}) \geq \min_{1 \leq l \leq m} \pi_i(\overline{x}, ..., x^k, ..., \overline{x}). \tag{67}$$

Thereby, $co(x^1, ..., x^m)$ is the convex hull of set $\{x^1, ..., x^m\}$.

For diagonal better-reply security, two further definitions from Reny (1999) are required:

**Definition 4.** (see Reny (1999), p. 1040) Player $i$ can **secure a payoff** of $\alpha \in \mathbb{R}$ **along the diagonal** at $(x, ..., x) \in X^N$ if there exists $\overline{x} \in X$ such that $\pi_i(x', ..., \overline{x}, ..., x') \geq \alpha$ for all $x'$ in some open neighborhood of $x \in X$.

**Definition 5.** (see Reny (1999), p. 1040) A game $\Gamma = (X_1, \pi_i)_{i=1}^N$ **is diagonally better reply secure** if whenever $(x^*, \pi^*) \in X \times \mathbb{R}$ is in the closure of the graph of its diagonal payoff function and $(x^*, ..., x^*)$ is not an equilibrium, some player $i$ can secure a payoff strictly above $\pi^*$ along the diagonal at $(x^*, ..., x^*)$.

Before we verify that these conditions are fulfilled in our game, we need the following Lemma.

**Lemma 16.** *Quasi-concavity of each player's payoff function in $(e_{ih}, e_{il}$ implies diagonal quasi-concavity of the game considered in this paper.*

*Proof.* Consider an arbitrary set $\{e_i^1, ..., e_i^m\} \in [0, \frac{sw}{c}]^2$ and an arbitrary $(\overline{e}_{ih}, \overline{e}_{il}) \in co\{e_i^1, ..., e_i^m\}$.

1. Case 1: $(\overline{e}_{ih}, \overline{e}_{il})$ is not a corner point of $co\{e_i^1, ..., e_i^m\}$, i.e. $(\overline{e}_{ih}, \overline{e}_{il})$ is a convex combination of two unequal points $(e_{ih}, e_{il})'$ and $(e_{ih}, e_{il})'' \in co\{e_i^1, ..., e_i^m\}$ whereby $(\overline{e}_{ih}, \overline{e}_{il}) \neq (e_{ih}, e_{il})'$ and $(\overline{e}_{ih}, \overline{e}_{il}) \neq (e_{ih}, e_{il})''$. Then, since the payoff function is quasi-concave in $(e_{ih}, e_{il})$, $\Pi(\overline{e}_{ih}, \overline{e}_{il}, \overline{e}_h, \overline{e}_l) \geq \min\Big(\Pi(e'_{ih}, e'_{il}, \overline{e}_h, \overline{e}_l), \Pi(e''_{ih}, e''_{il}, \overline{e}_h, \overline{e}_l)\Big)$. By transitivity of the real numbers,
$$\Pi(\overline{e}_{ih}, \overline{e}_{il}, \overline{e}_h, \overline{e}_l) \geq \min_{1 \leq k \leq m} \Pi(e_{ih}^k, e_{il}^k, \overline{e}_h, \overline{e}_l). \tag{68}$$

2. Case 2: $(\overline{e}_{ih}, \overline{e}_{il})$ is a corner point of $co\{e_i^1, ..., e_i^m\}$, i.e. it cannot be expressed as a convex combination of two distinct other points of the set. Then $(\overline{e}_{ih}, \overline{e}_{il}) \in \{e_i^1, ..., e_i^m\}$. Hence,
$$\Pi(\overline{e}_{ih}, \overline{e}_{il}, \overline{e}_h, \overline{e}_l) \geq \min_{1 \leq k \leq m} \Pi(e_{ih}^k, e_{il}^k, \overline{e}_h, \overline{e}_l). \tag{69}$$

■

The theorem by Reny (1999) states sufficient conditions for the existence of at least one equilibrium. Since we have already proven that the case where all players of all types play zero networking effort is always a PBE, we will now modify our game such that we can prove that the original game has at least one further symmetric PBE, given the restrictions specified in Proposition 2.

Consider a game that is identical to the original game, except for the following modification. For both types of workers $p \in \{h, l\}$, the diagonal payoff function is

$$\overline{\Pi}_{ip}(e_{ip}, e_h, e_l) = \begin{cases} -w & \text{if } e_{ip} = e_h = e_l = 0 \\ \Pi_{ip}(e_{ip}, e_h, e_l) & \text{otherwise} \end{cases} \tag{70}$$

Note that $\Pi_{ip}(e_{ip}, e_h, e_l)$ is the original function as specified in (26). This modification eliminates the zero-effort equilibrium of the original game, because starting from a situation in which any player plays zero effort, every worker has an incentive to deviate to a strictly positive effort level close to zero, which yields a strictly positive expected payoff (the expected income from not losing the job plus the expected income from a job found via the formal labor market, minus a very small networking cost). Since also the original payoff function is discontinuous when $e_h = 0$ and $e_l = 0$, the modification does not, however, eliminate any further equilibria and does not create any new equilibria. Since the point where all workers play zero-effort cannot be reached by a unilateral deviation from any other symmetric strategy profile, the elimination of further equilibria is excluded. On the other hand, whenever a deviation from a symmetric strategy profile to zero effort of the deviator is beneficial in the original game, it is still beneficial in the modified game. Hence, no new equilibria are created.

The following analysis relies on the "ex-ante"-definition of a PBE (see Vega-Redondo (2003), p. 197 for a detailed introduction).[6] Players decide on their strategies before nature decides on their types, hence their strategies correspond to tuples $e_i = (e_{ih}, e_{il})$ and their payoff function is the ex-ante expected payoff, i.e. the sum of the payoff functions of both types of the player, weighted with the probability that they are of either type. We denote the diagonal ex-ante payoff function of player $i$ by

$$\tilde{\Pi}_i(e_{ih}, e_{il}, e_h, e_l) = r\tilde{\Pi}_i(e_{ih}, e_h, e_l) + (1-r)\tilde{\Pi}_i(e_{il}, e_h, e_l) \tag{71}$$

The following lemma shows that the requirements of Reny (1999) are fulfilled for the modified game.

**Lemma 17.** *The modified game is (1) compact and (2) quasi-symmetric. If conditions* (34) *hold, the game is also (3) diagonal quasi-concave and (4) diagonally better-reply secure.*

*Proof.*

1. As we have restricted the strategy space to the interval $[0, sw/c]$ and $\tilde{\Pi}_i(e_{ih}, e_{il}, e_h, e_l)$ takes on finite values for all $(e_{ih}, e_{il}, e_h, e_l) \in [0, sw/c]^4$, the modified game is clearly a compact game.

2. Since the strategy sets are identical for each player and the diagonal payoff functions $\tilde{\Pi}(e_{ih}, e_{il}, e_h, e_l)$ are equal for all players and do not depend on the player's identity, the game is quasi-symmetric.

3. If $\gamma_l < 1/2$, by Lemmata 8 and 2, the interim payoff function of high- and low-productivity workers is concave in $e_{ih}$ and $e_{il}$ respectively. As these functions are constant in the own effort of the other type, both functions are also concave in $(e_{ih}, e_{il})$. Thus, the ex-ante payoff function of worker $i$, $r\tilde{\Pi}_i(e_{ih}, e_h, e_l) + (1-r)\tilde{\Pi}_i(e_{il}, e_h, e_l)$, is a weighted sum of two concave functions and is therefor also concave in $(e_{ih}, e_{ih})$. Hence, if $\gamma < 1/2$, Lemmata 8 and 16 ensure that the game is diagonally quasi-concave.

4. As the payoff functions are continuous anywhere except for the point where $e_h = 0$ and $e_l = 0$, diagonal better reply security needs to be checked at this point. The diagonal utility functions of the modified game are discontinuous at $e_i = (0, 0)$. For each type of the player, there are two payoff-values in the closure of the graph of this utility function, $-w$ and $(1-s)w + s\psi^0 w$. Condition (34) ensures that each player can secure a payoff of $(1-s)w + s\psi^0 w$ along the diagonal at $e_i = (0, 0)$ by deviating to effort level $e_i = (\lim_{e_h \to 0} BR_h(e_h, 0), \lim_{e_l \to 0} BR_l(0, e_l))$.

■

[6] Note that this definition corresponds in technical respects to the interim definition of PBE used elsewhere in the paper.

We now proceed with the proof of proposition 2.

*Proof.* Suppose (34) holds. By Lemma 17 all requirements of Reny (1999) are fulfilled for the modified game. Thus, it has at least one equilibrium. Since the modification has eliminated the PBE in which all players of all types play zero effort without creating any new equilibria, it follows that the original game has at least one equilibrium in which at least one type of the players play a strictly positive effort level. ■

**Proof of Lemma 9**

*Proof.*

1. (a) $\partial\epsilon_{ih}(e_{ih}, e_h, 0)/\partial e_{ih} = -\exp[e_{ih} f(e_h, 0)] f[e_h, 0]$. Since $f(e_h, 0) < 0$, this derivative is strictly positive for all $e_h > 0$.

   (b) $\partial\epsilon_{ih}(e_{ih}, e_h, 0)/\partial e_h = -\exp[e_{ih} f(e_h, 0)] e_{ih} \cdot \partial f(e_h, 0)/\partial e_h$.
   $\partial f(e_h, 0)/\partial e_h = (\exp[-e_h s](-1 + \exp[e_h s] - e_h s)\theta) e_h^{-2} > 0$, because $\lim_{e_h \to 0}(-1 + \exp[e_h s] - e_h s) = 0$ and $\partial(\exp[e_h s] - e_h s)/\partial(e_h s) = \exp[e_h s] - 1 > 0 \; \forall \; e_h s > 0$.
   Hence, $\partial\epsilon_{ih}(e_{ih}, e_h, 0)/\partial e_h < 0$.

2. $\partial\epsilon_h(e_h, 0)/\partial e_h = \exp[e_h f(e_h, 0)](s\theta \exp[-e_h s]) > 0$.

■

**Proof of Lemma 10**

*Proof.*

1. $\vartheta(e_h, 0) \leq 1 \Longleftrightarrow \theta - r\epsilon_h(e_h, 0) \geq 1 - r\epsilon_h(e_h, 0) \Longleftrightarrow \theta \leq 1$.

2. $\vartheta(e_h, 0) \geq \theta \Longleftrightarrow \theta - r\epsilon_h(e_h, 0) \geq \theta(1 - r\epsilon_h(e_h, 0)) \Longleftrightarrow r\epsilon_h(e_h, 0) \leq \theta(r\epsilon_h(e_h, 0)) \Longleftrightarrow \theta \geq 1$. ■

**Proof of Lemma 11**

*Proof.* Observe that $\partial f(e_h, 0)/\partial e_h = (\exp[-e_h s](-1 + \exp[e_h s] - e_h s)\theta) e_h^2 > 0$, because $\lim_{e_h \to 0}(-1 + \exp[e_h s] - e_h s) = 0$ and $\partial(\exp[e_h s] - e_h s)/\partial(e_h s) = \exp[e_h s] - 1 > 0 \; \forall \; e_h s > 0$. Observe furthermore that by the chain rule $\partial\vartheta(e_h, 0)\partial e_h = \partial\vartheta/\partial\epsilon_h \cdot \partial\epsilon_h/\partial e_h$. Hence,

$$\frac{\partial\vartheta(e_h, 0)}{\partial e_h} \geq 0 \Longleftrightarrow \frac{\partial\vartheta}{\partial\varepsilon_h} \geq 0 \Longleftrightarrow -r\frac{1-\theta}{(1-r\epsilon_h)^2} \geq 0 \Longleftrightarrow \theta \geq 1. \tag{72}$$

■

**Proof of Lemma 12**

*Proof.* In any separating equilibrium, $BR_h(e_h, 0) = e_h$. Observe that $c/(s^2 w) < \theta(1 - \psi^0)$ implies

$$\lim_{e_h \to 0} BR_h(e_h, 0) = \left(\ln\left[\frac{s^2\theta w}{c}(1 - \psi^0)\right]\right)/(s\theta) > 0 = \lim_{e_h \to 0} e_h \tag{73}$$

The graph of $e_h$ is monotonically increasing in $e_h \; \forall \; e_h > 0$. Hence, it suffices to show that $\lim_{e_h \to \infty} BR_h(e_h, 0) = -\infty$ in order to prove that the graph of $BR_h(e_h, 0)$ intersects the graph of $e_h$ at least once.
Observe that $\lim_{e_h \to \infty}(-f(e_h, 0)) = 0$ and

$$0 < \lim_{e_h \to \infty}(1 - \psi(e_h, 0)) = 1 - \psi\left(\frac{\theta - r(1 - \exp[-\theta])}{1 - r(1 - \exp[-\theta])}\right) < 1. \tag{74}$$

Hence,

$$\lim_{e_h \to \infty} \ln \left[ \frac{sw}{c} (f(e_h, 0))(1 - \psi(e_h, 0)) \right] = -\infty. \tag{75}$$

and therefore also

$$\lim_{e_h \to \infty} BR_h(e_h, 0) = -\infty. \tag{76}$$

■

**Proof of Proposition 3**

*Proof.* By Lemma 12, if $c/(s^2 w) < \theta(1-\psi^0)$, there exists at least one $e_h^*$ such that $BR_h(e_h^*, 0) = e_h^*$. A sufficient condition for $BR_l(e_h^*, 0) = 0$ is $\gamma_l \geq (1 - \psi(e_h^*, 0))/(\phi(e_h^*, 0)) = \overline{\gamma}(\vartheta(e_h^*, 0))$.
By Lemma 2, $\overline{\gamma}(\vartheta(e_h, 0)) \in ]0.5, 1[\ \forall\ e_h > 0$. Hence, there exists a $\overline{\gamma}^S$ such that $\gamma_l > \overline{\gamma}^S$ implies that a symmetric strategy profile where all high-productivity workers play $e_h = e_h^*$ and all low-productivity workers play $e_l = 0$ constitutes a symmetric separating equilibrium.
Suppose now $\theta \geq 1$. Then, by Lemma 11, $\lim_{e_h \to \infty} \vartheta(e_h, 0) > \vartheta(e_h^*, 0)$, and by Lemma 2 combined with Lemma 11, $\overline{\gamma}(\vartheta(e_h, 0))$ is monotonically decreasing in $e_h$. Hence, $\overline{\gamma}(\vartheta(e_h^*, 0)) < (1 - \psi(\vartheta))/\phi(\vartheta)$.
Thus, $\gamma_l > (1 - \psi(\vartheta))/\phi(\vartheta) \Rightarrow \gamma_l > \overline{\gamma}(\vartheta(e_h^*, 0))$.
Finally, suppose $\theta < 1$. Then by Lemma 11, $\lim_{e_h \to \infty} \vartheta(e_h, 0) > \vartheta(e_h^*, 0)$, and by Lemma 2 combined with Lemma 11, $\overline{\gamma}(\vartheta(e_h, 0))$ is monotonically increasing in $e_h$.
Hence, $\overline{\gamma}(\theta(e_h^*, 0)) < (1 - \psi(\lim_{e_h \to \infty} \vartheta(e_h, 0)))/\phi(\lim_{e_h \to 0} \vartheta(e_h, 0))$.
Thus, $\gamma_l > (1 - \psi(\lim_{e_h \to \infty} \vartheta(e_h, 0)))/\phi(\lim_{e_h \to 0} \vartheta(e_h, 0)) \Rightarrow \gamma_l > \overline{\gamma}(\vartheta(e_h^*, 0))$. ■

**Proof of Proposition 4**

*Proof.* Consider

1. **High-productivity workers:** The probability that an unemployed high-productivtiy worker finds a job in a zero-effort equilibrium is equal to $\psi^0 = \psi(\theta)$. The probability that he finds a job in a separating equilibrium is $\rho(e_h^*, 0)$.

   (a) Suppose that $\theta > 1$. Then, by Lemma 10, $\vartheta(e_h, 0) > \theta$. Since $\partial\psi(\vartheta)\partial\vartheta > 0$, it follows that $\psi(e_h, 0) > \psi^0\ \forall\ e_h > 0$. As $1 - \psi(e_h, 0) > 0$ and $\epsilon_h(e_h, 0) > 0\ \forall\ e_h > 0$, it follows that $\rho(e_h, 0) > \psi^0\ \forall\ e_h > 0$ and hence for any $e_h^* > 0$.

   (b) Suppose that $\theta < 1$. Then by Lemma 10, $\vartheta(e_h, 0) < \theta$.
   We will first prove that $\rho(e_h, 0)$ is bounded from below by $\tilde{\rho}(e_h, 0) \equiv \lim_{r \to 1} \rho(e_h, 0)$ for each $e_h > 0$. Then we will show that the function $\rho(e_h, 0)$ is strictly increasing in $e_h$ for any $e_h > 0$. Provided this is true, it is sufficient to show that the limit of $\tilde{\rho}(e_h, 0)$ for $e_h \to 0$ is at least as large as $\psi^0$.
   Consider the derivative $\partial\theta(e_h, 0)/\partial r = -\epsilon_h(e_h, 0)(1 - \theta)(1 - r\epsilon_h(e_h, 0))^{-2}$. $\theta < 1$ implies that $\vartheta(e_h, 0)$ is decreasing in $r$. Hence, the family of functions $\vartheta(e_h, 0)$ for different $r$ is bounded from below by $\lim_{r \to 1} \vartheta(e_h, 0) = (\theta - \epsilon_h(e_h, 0))/(1 - \epsilon_h(e_h, 0)) \equiv \tilde{\vartheta}$.
   As the derivative of $\psi(\vartheta)$ with respect to $\vartheta$, i.e. $\partial\psi(\vartheta)\partial\vartheta = 1 - \exp[-1/\vartheta](1 + \vartheta)\vartheta^{-1} > 0$, and since $\epsilon_h(e_h, 0)$ is independent of $r$, the expression $\rho(e_h, 0) = \epsilon_h(e_h, 0) + (1 - \epsilon(e_h, 0))\psi(\vartheta(e_h, 0))$ is decreasing in $r$.
   Hence, $\rho(e_h, 0)$ is bounded from below by

$$\tilde{\rho}(e_h, 0) = \lim_{r \to 1} \rho(e_h, 0) = \epsilon_h(e_h, 0) + (1 - \epsilon_h(e_h, 0))\psi(\tilde{\vartheta}) \tag{77}$$

for each $e_h \geq 0$.

Next, consider the first derivative of $\tilde{\rho}(e_h, 0)$ with respect to $e_h$. By the chain rule,

$$\begin{aligned} \partial\tilde{\rho}(e_h,0)/\partial e_h &= \partial\epsilon_h(e_h,0)/\partial e_h \cdot (1-\psi(\tilde{\vartheta}) + (1-\epsilon_h(e_h,0)) \cdot \partial\psi(\tilde{\vartheta})/\partial\tilde{\vartheta} \cdot \partial\tilde{\vartheta}/\partial e_h) \\ &= \frac{\partial\epsilon_h(e_h,0)}{\partial e_h}\left(\frac{1-\epsilon_h(e_h,0)}{\theta-\epsilon_h(e_h,0)}\exp\left[-\frac{1-\epsilon_h(e_h,0)}{\theta-\epsilon_h(e_h,0)}\right]\right) \end{aligned} \tag{78}$$

By Lemma 9, $\partial\epsilon_h(e_h, 0)/\partial e_h > 0$ and by Lemma 5, $\theta > \epsilon_h(e_h, 0)$. It follows that $\partial\tilde{\rho}(e_h, 0)/\partial e_h > 0$. Since $\lim_{e_h \to 0} \tilde{\rho}(e_h, 0) = \psi^0$, it follows that $\rho(e_h) > \psi^0 \; \forall \; 0 < r < 1$ and $\forall \; e_h > 0$.

2. **Low-productivity workers:** The probability that a low-productivity worker finds a job in any separating equilibrium is equal to $\psi(\vartheta(e_h^*, 0))$, where $e_h^*$ is the respective equilibrium effort level of a high-productivity worker. The probability that a low-productivity worker finds a job in a co-existing zero-effort equilibrium is equal to $\psi^0 = \psi(\theta)$. By Lemma 1, $\partial\psi(\vartheta)/\partial\vartheta > 0$. By Lemma 5, $\vartheta(\epsilon_h(e_h, 0), 0) \geq \theta$ is equivalent to $\theta \geq 1$. It follows that $\psi(\vartheta(e_h, 0)) \geq \psi^0 \iff \theta \geq 1$.

3. The probability that a filled vacancy is filled by a high-productivity worker is $r$ in the zero-effort equilibrium and

$$\frac{r[\epsilon_h(e_h^*,0) + (1-\epsilon_h(e_h^*,0))\psi(e_h^*,0)]}{r[\epsilon_h(e_h^*,0) + (1-\epsilon_h(e_h^*,0))\psi(e_h^*,0)] + (1-r)\psi(e_h^*,0)} \tag{79}$$

$$= \; r \cdot \frac{\psi(e_h^*,0) + (1-\psi(e_h^*,0))\epsilon_h(e_h^*,0)}{\psi(e_h^*,0) + (1-\psi(e_h^*,0))r\epsilon_h(e_h^*,0)} > r. \tag{80}$$

in any separating equilibrium equilibrium with $e_i = e(e_h^*, 0) \; \forall \; i \in \mathcal{M}$ and $e_h^* > 0$.

■

### Proof of Lemma 13 (CHECK!)

*Proof.* Suppose $\gamma_l \geq \gamma_h$. Then, $\frac{1-\gamma_l}{1-\gamma_h} > 1$. Suppose $e_h > e_l > 0$.

$$\begin{aligned} \frac{g_{jh}\alpha_h(1-\epsilon_l)}{g_{jl}\alpha_l(1-\epsilon_h)} &= \frac{e_h}{e_l}\left(\frac{1-\exp[-e_h s]}{e_h s} \Big/ \frac{1-\exp[-e_l s]}{e_l s}\right)\left(\frac{\exp[e_h f(e_h,e_l)]}{\exp[e_l f(e_h,e_l)]}\right) \\ &= \frac{\exp[e_l f(e_h,e_l)] - \exp[-e_h e_l s f(e_h,e_l)]}{\exp[e_h f(e_h,e_l)] - \exp[-e_h e_l s f(e_h,e_l)]}. \end{aligned} \tag{81}$$

As $f(e_h, e_l) < 0$, it follows from $e_l < e_h$ that $\frac{g_{jh}\alpha_h(1-\epsilon_l)}{g_{jl}\alpha_l(1-\epsilon_h)} < 1$. Hence, inequality (43) is fulfilled. ■

### Proof of Lemma 15

*Proof.* For $e_h > 0$ and $e_l > 0$, $BR_{ih}(e_h, e_l) - BR_{il}(e_h, e_l) = \{\ln[1 - \psi(e_h, e_l)] - \ln[1 - \psi(e_h, e_l) - \gamma_l\phi(e_h, e_l)]\}/[-f(e_h, e_l)] = \ln[(1 - \psi(e_h, e_l))/(1 - \psi(e_h, e_l) - \gamma_l\phi(e_h, e_l))]/[-f(e_h, e_l)]$. As $(1 - \psi(e_h, e_l))/(1 - \psi(e_h, e_l) - \gamma_l\phi(e_h, e_l)) > 1$, the term $\ln[(1 - \psi(e_h, e_l))/(1 - \psi(e_h, e_l) - \gamma_l\phi(e_h, e_l))]$ is positive and so is $-f(e_h, e_l)$. Hence, $BR_{ih}(e_h, e_l) > BR_{il}(e_h, e_l)$. ■

### Proof of Proposition 5

*Proof.* Consider a symmetric pooling equilibrium. Suppose $e_h^*$ and $e_l^*$ are the corresponding equilibrium effort levels. $\rho_p(e_h^*, e_l^*)$ is the probability that an unemployed worker of type $p \in \{h, l\}$ finds a job in this equilibrium. The probability that an unemployed worker of any type finds a job in the co-existing zero-effort equilibrium is equal to $\psi^0 = \psi(\theta)$. By Lemma 15, $e_h^* > e_l^*$ and therefore $\epsilon_h(e_h^*, e_l^*) > \epsilon_l(e_h^*, e_l^*)$.

1. The probability that a filled vacancy is occupied by a high-productivity worker is $r$ in the zero-effort equilibrium and

$$\frac{r\rho_h(e_h^*, e_l^*)}{r\rho_h(e_h^*, e_l^*) + (1-r)\rho_l(e_h^*, e_l^*)} \tag{82}$$

$$= r \cdot \frac{\psi(e_h^*, e_l^*) + (1-\psi(e_h^*, e_l^*))\epsilon_h(e_h^*, e_l^*)}{\psi(e_h^*, e_l^*) + (1-\psi(e_h^*, e_l^*))(r\epsilon_h(e_h^*, e_l^*) + (1-r)\epsilon_l(e_h^*, e_l^*)) - (1-r)\gamma\phi(e_h^*, e_l^*)\epsilon_l(e_h^*, e_l^*)} \tag{83}$$

in any pooling equilibrium with $e_i = (e_h^*, e_l^*)\ \forall\ i \in \mathcal{M}$ and $e_h^* > 0$ and $e_l^* > 0$.

2. The overall unemployment rate in a symmetric pooling equilibrium is $s(1 - r\rho_h(e_h^*, e_l^*) - (1-r)\rho_l(e_h^*, e_l^*))$. The overall unemployment rate in the zero-effort equilibrium is $s(1-\psi^0)$.

   (a) Suppose $(e_h^*, e_l^*)$ is such that $\vartheta(e_h^*, e_l^*) < \theta$. Then, $r\rho_h(e_h^*, e_l^*) + (1-r)\rho_l(e_h^*, e_l^*) = r\epsilon(e_h^*, e_l^*) + (1-r)(1-\gamma_l\phi(e_h^*, e_l^*))\epsilon_l(e_h^*, e_l^*) + (1 - r\epsilon_h - (1-r)\epsilon_l)\psi(e_h^*, e_l^*) = r\epsilon_h(e_h^*, e_l^*) + (1-r)(1-\gamma_\phi(e_h^*, e_l^*))\epsilon_l(e_h^*, e_l^*) + (1 - r\epsilon_h - (1-r)\epsilon_l)\vartheta(e_h^*, e_l^*)\phi(e_h^*, e_l^*) = r\epsilon_h(e_h^*, e_l^*) + (1-r)(1-\gamma_l\phi(e_h^*, e_l^*))\epsilon_l(e_h^*, e_l^*) + (\theta - r\epsilon_h - (1-r)(1-\gamma_l)\epsilon_l)\phi(e_h^*, e_l^*)$

$$\underbrace{(1-\phi(e_h^*, e_l^*))(r\epsilon_h(e_h^*, e_l^*) + (1-r)\epsilon_l(e_h^*, e_l^*))}_{>0} + \underbrace{\theta\phi(e_h^*, e_l^*)}_{>\theta\phi^0 = \psi^0} > \psi^0$$

   Hence, $s(1 - r\rho_h(e_h^*, e_l^*) + (1+r)\rho_l(e_h^*, e_l^*)) < s(1-\psi^0)$.

   (b) Suppose now $(e_h^*, e_l^*)$ is such that $\vartheta(e_h^*, e_l^*) \geq \theta$. Then, $\psi(e_h^*, e_l^*) \geq \psi^0$. Hence,

$$\rho_h(e_h^*, e_l^*) = \psi(e_h^*, e_l^*) + (1-\psi(e_h^*, e_l^*))\epsilon_h(e_h^*, e_l^*) > \psi^0 \tag{84}$$

   and

$$\rho_l(e_h^*, e_l^*) = \psi(e_h^*, e_l^*) + \underbrace{(1-\psi(e_h^*, e_l^*) - \gamma_l\phi(e_h^*, e_l^*))}_{>0 \text{ by (19) in equilibrium}}\epsilon_l(e_h^*, e_l^*) > \psi^0 \tag{85}$$

   Thus, $s(1 - r\rho_h(e_h^*, e_l^*) - (1-r)\rho_l(e_h^*, e_l^*)) < s(1-\psi^0)$.
   As the $r\rho_h(e_h^*, e_l^*) + (1-r)\rho_l(e_h^*, e_l^*)$ is a convex combination of $\rho_h(e_h^*, e_l^*)$ and $\rho_l(e_h^*, e_l^*)$, and $\rho_h(e_h^*, e_l^*) > \rho_l(e_h^*, e_l^*)$, $\rho_h(e_h^*, e_l^*) > \psi^0$. Hence, $s(1 - \rho_h(e_h^*, e_l^*) < s(1-\psi^0)$.

If $\theta \geq 1$, then $\vartheta(e_h^*, e_l^*) > \theta$. By (84) and (85), the probability to find a job for both types are higher in any pooling equilibrium than in the co-existing zero-effort equilibrium. Therefore, the corresponding unemployment rates among high- and low-productivity workers are lower in any pooling equilibrium than in the co-existing zero-effort equilibrium.

■